Rate and uncertainty of the $^{18}Ne(\alpha,p)^{21}Na$ reaction from Monte-Carlo simulations

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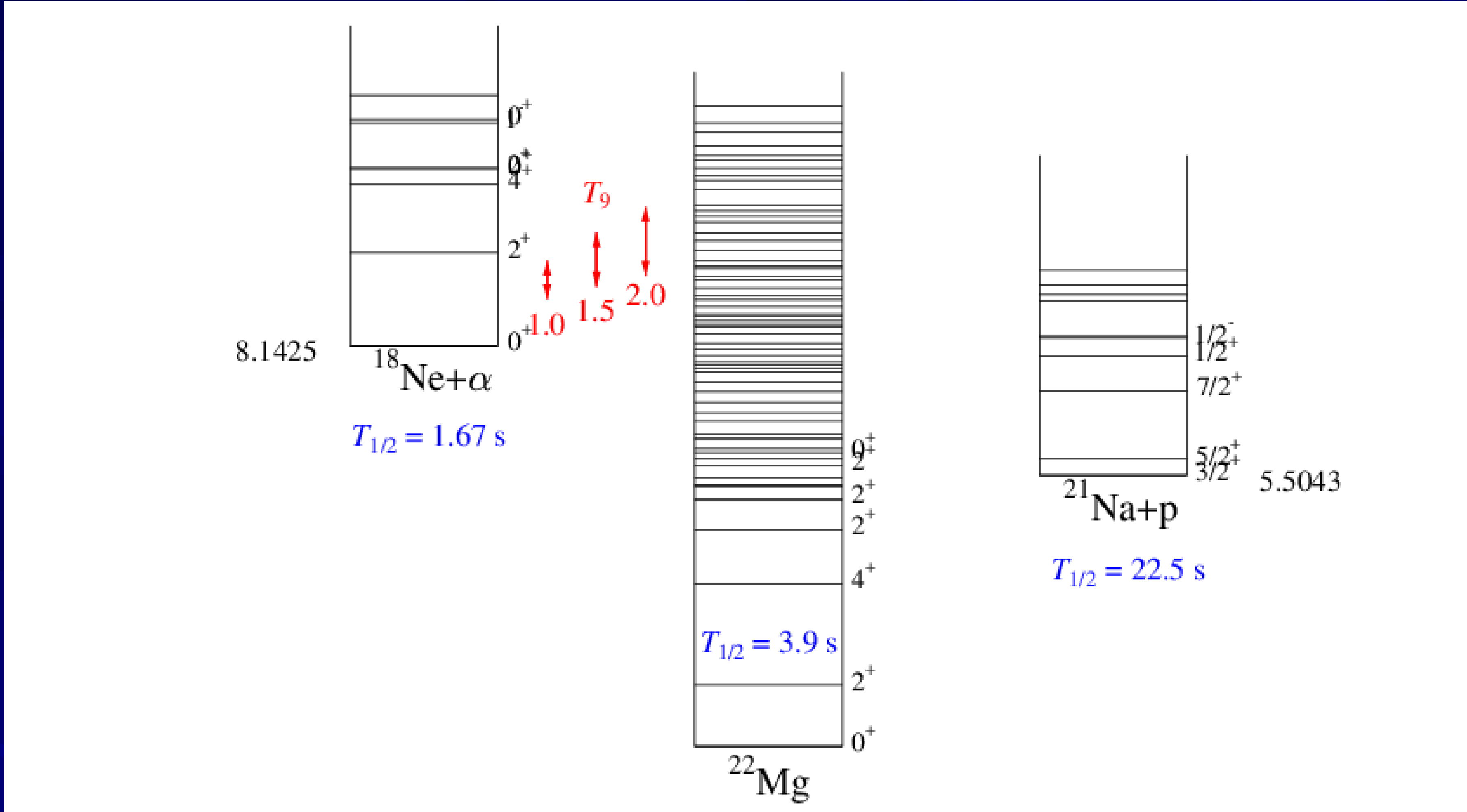
Nuclei in the Cosmos - Debrecen 2014

Motivation

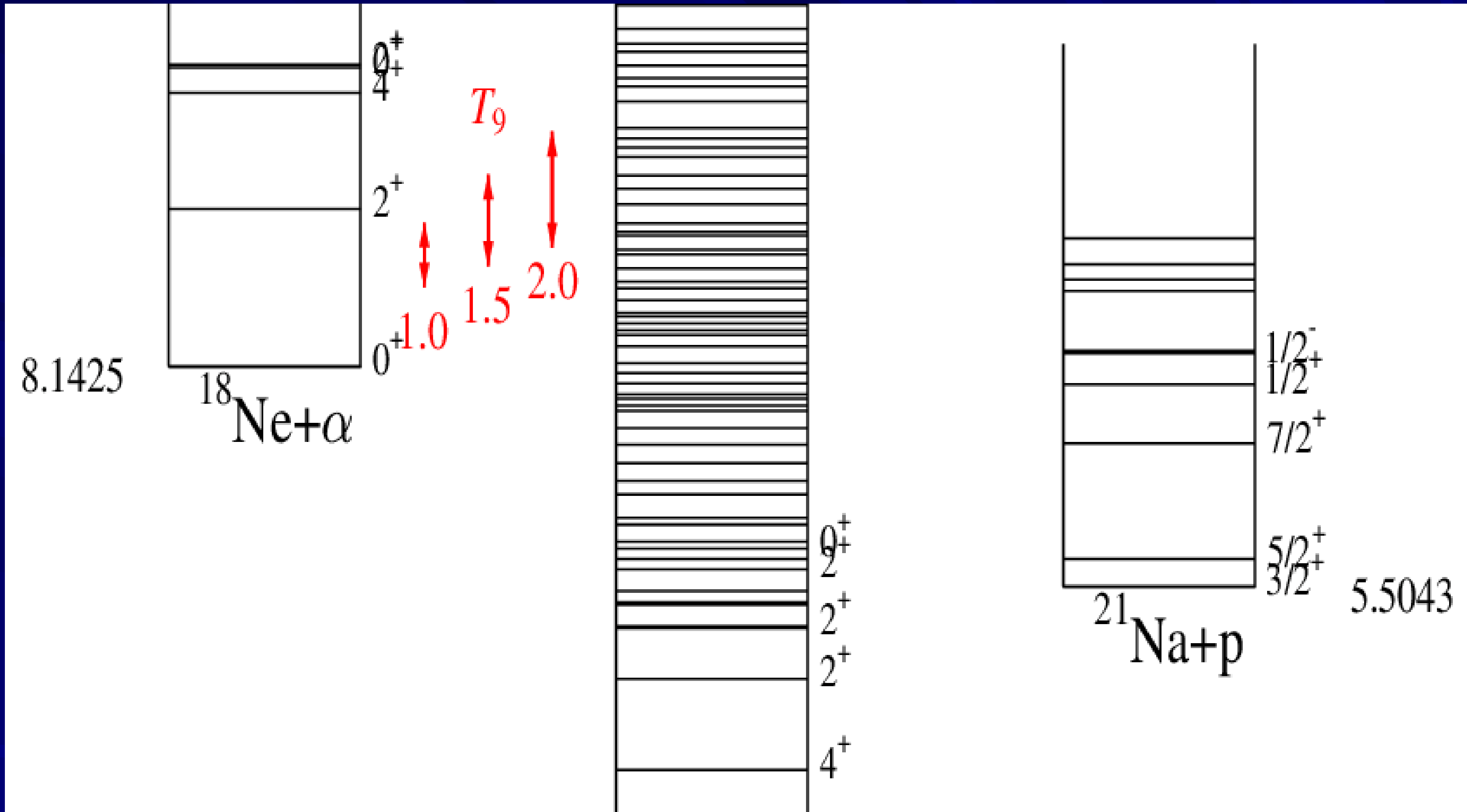
- (see previous talks)
- In very brief:
  - break-out from hot CNO cycles to the rp-process
  - astrophysical site: X-ray bursters (XRB)
  - relevant temperature range: $0.5 \leq T_9 \leq 2.0$
  - breakout reactions:
    - ^{18}Ne(α,p)^{21}Na: dominant for lower temperatures
    - ^{15}O(α,γ)^{19}Ne

A simplified level scheme of ^{22}Mg



- Reaction rate factor $N_A < \sigma v >$ of ^{18}Ne(α,p)^{21}Na dominated by resonances in ^{22}Mg at $E_x \approx$ 9 - 11 MeV

A simplified level scheme of ^{22}Mg (zoom)



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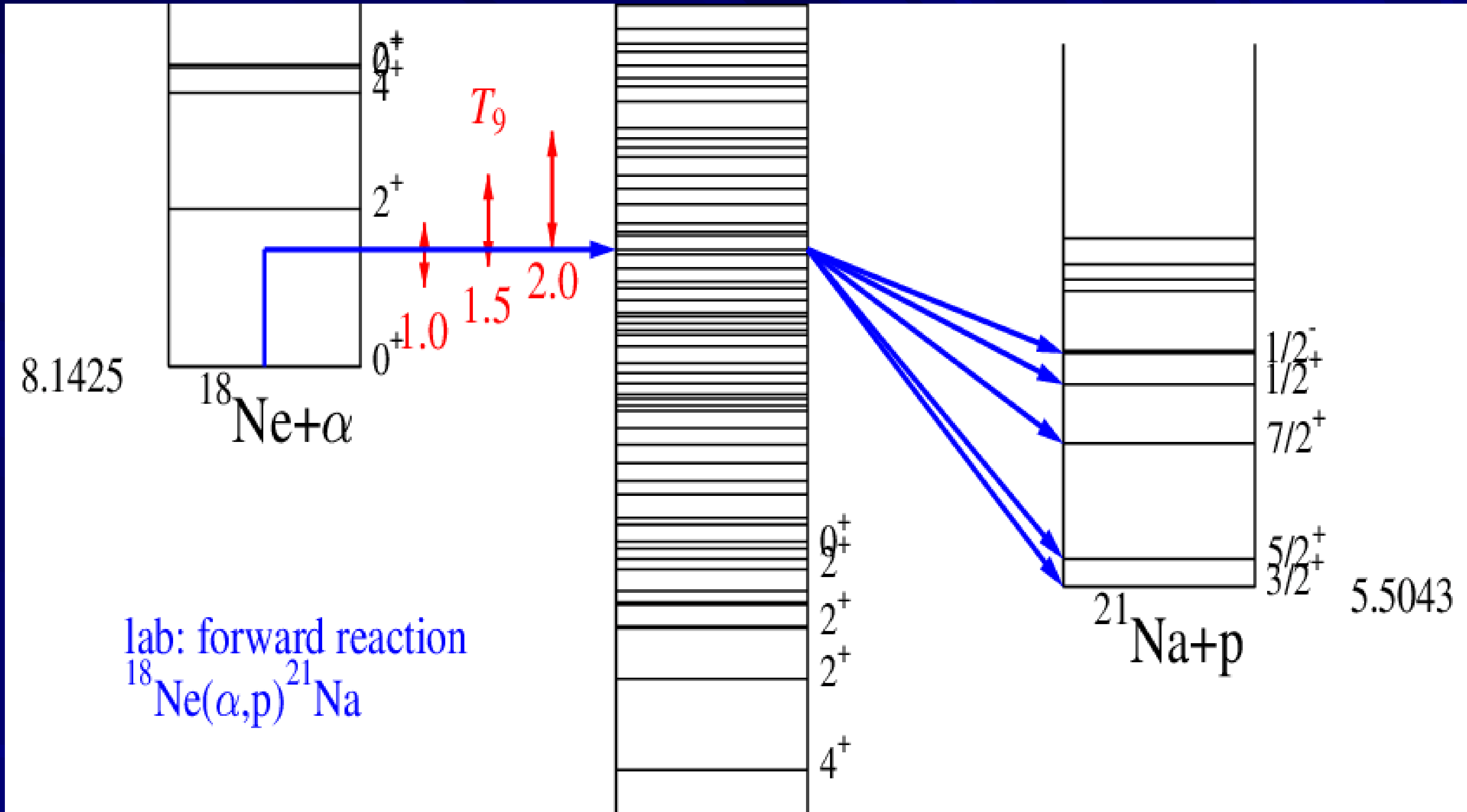
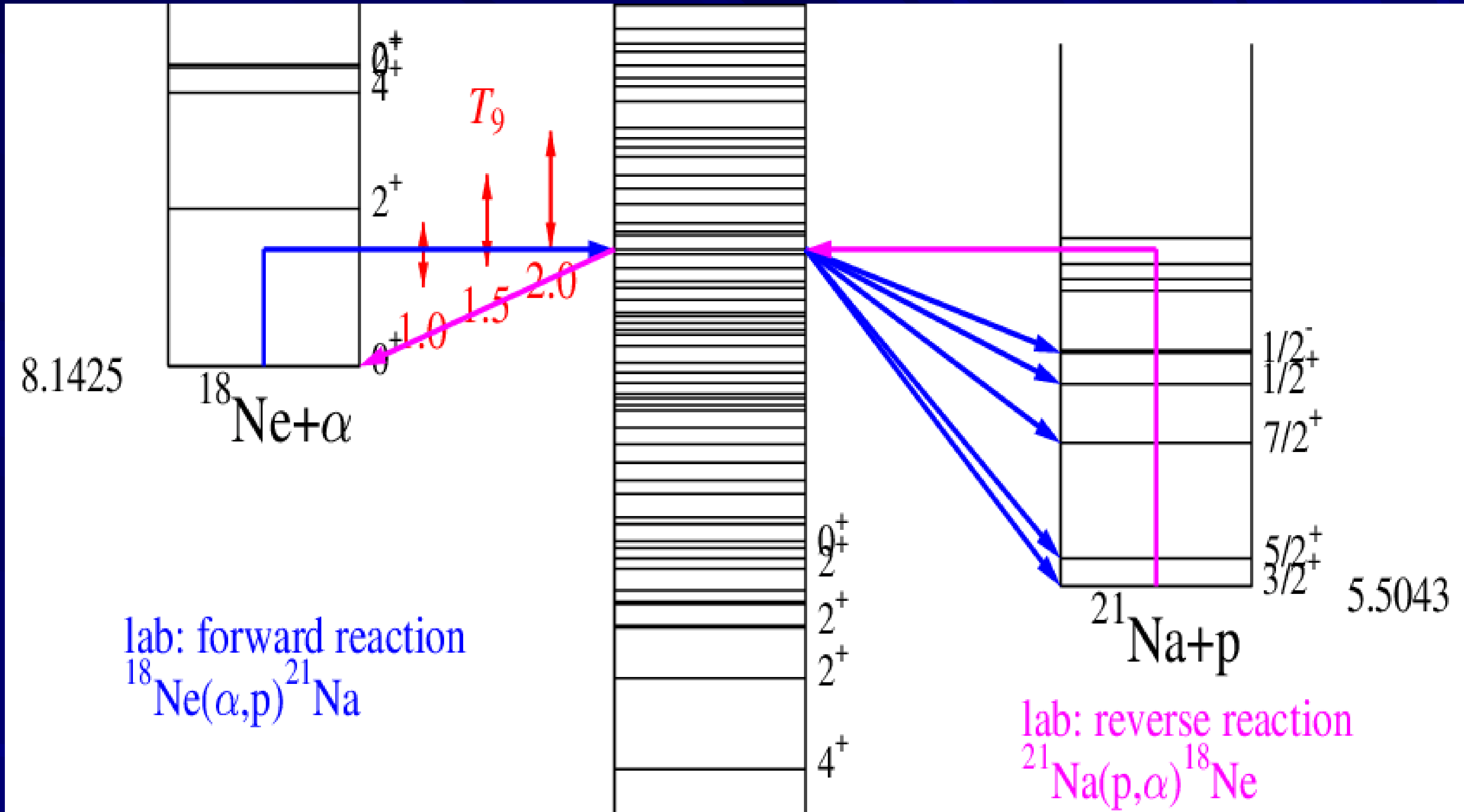
A simplified level scheme of ^{22}Mg (zoom)



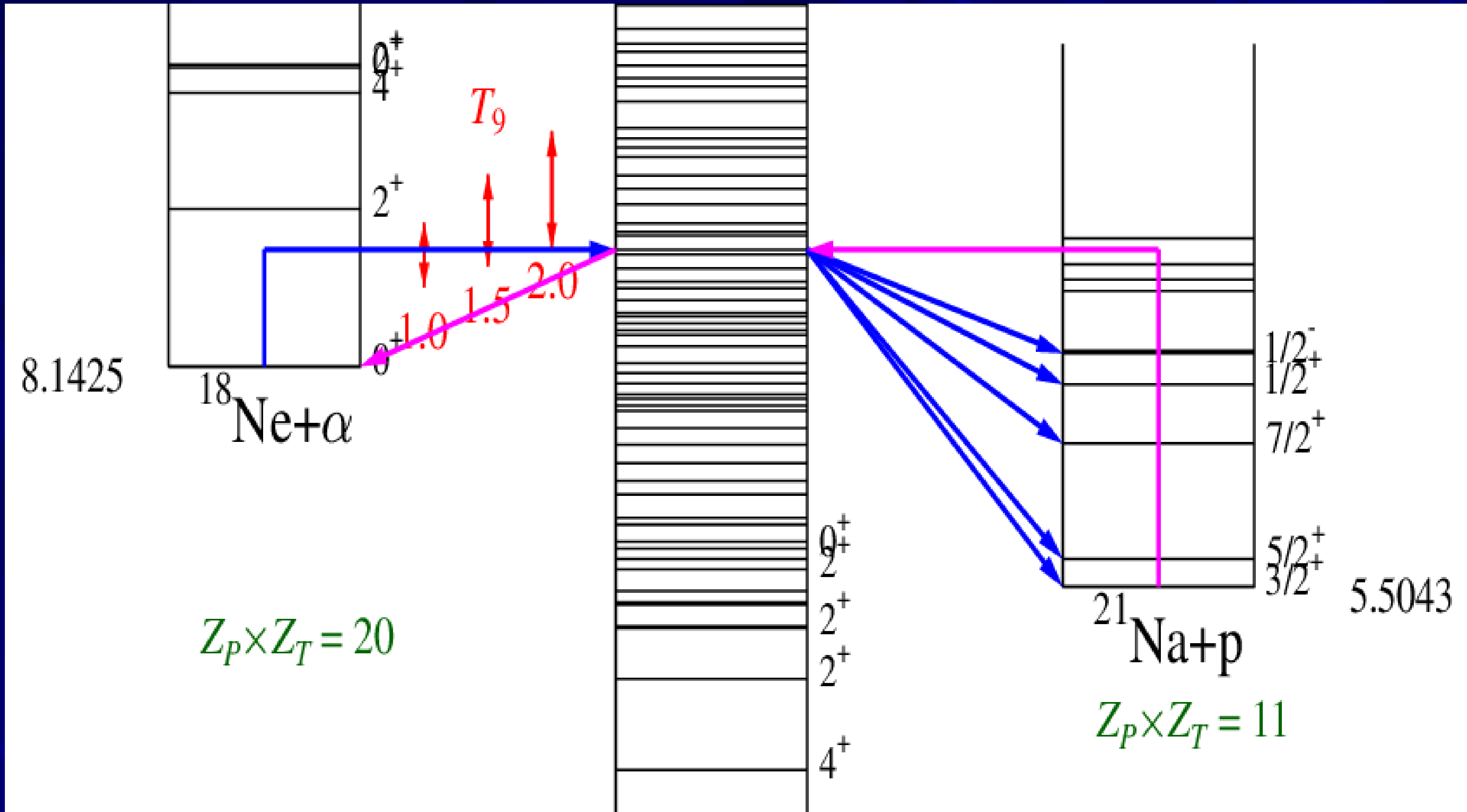
- Forward reaction ^{18}Ne(α,p)^{21}Na: contributions of several final states in ^{21}Na

A simplified level scheme of ^{22}Mg (zoom)



- Reverse reaction ^{21}Na(p,α)^{18}Ne: dominating final ground state in ^{18}Ne

A simplified level scheme of ^{22}Mg (zoom)



- Larger Coulomb barrier in ^{18}Ne+α channel → $\Gamma_\alpha \ll \Gamma_p$: proton decay of ^{22}Mg* faster than α decay

Approaches to determine the stellar rate

- **Direct approach:**
  measure ^{18}Ne(α,p)^{21}Na (in inverse kinematics)
  Groombridge *et al*., Phys. Rev. C **66**, 055802 (2002); Blackmon, NIC-XIII?
- **Indirect approach 1:** (also used in the present study)
  measure level properties in ^{22}Mg (E_x, Γ, J^π)
  calculate $N_A < \sigma v >$ from theoretical ωγ and Γ_α
  Matic *et al*., Phys. Rev. C **80**, 055804 (2009); Mohr and Matic, Phys. Rev. C **87**, 035801 (2013); Zhang *et al*., Phys. Rev. C **89**, 015804 (2014)
- **Indirect approach 2:** reverse reaction
  measure ^{21}Na(p,α)^{18}Ne (in inverse kinematics)
  use theoretical corrections for excited states
  Salter *et al*., Phys. Rev. Lett. **108**, 242701 (2012)
  Sinha, Rehm, *et al*., ANL Annual Report 2005 (unpublished)

Resonance strength

- Resonance strength ωγ:

$$\omega\gamma = (2J+1)\,\frac{\Gamma_\alpha \Gamma_p}{\Gamma} \approx (2J+1)\,\Gamma_\alpha$$

- Partial width Γ_α for the α-channel:

$$\Gamma_{\alpha,L} = \theta_\alpha^2 \times \Gamma_{\alpha,L}^{s.p.}(E) \;=\; \theta_\alpha^2 \times \frac{2kR}{F_L^2(E,R) + G_L^2(E,R)} \times \frac{\hbar^2}{\mu R^2}$$

depends on:

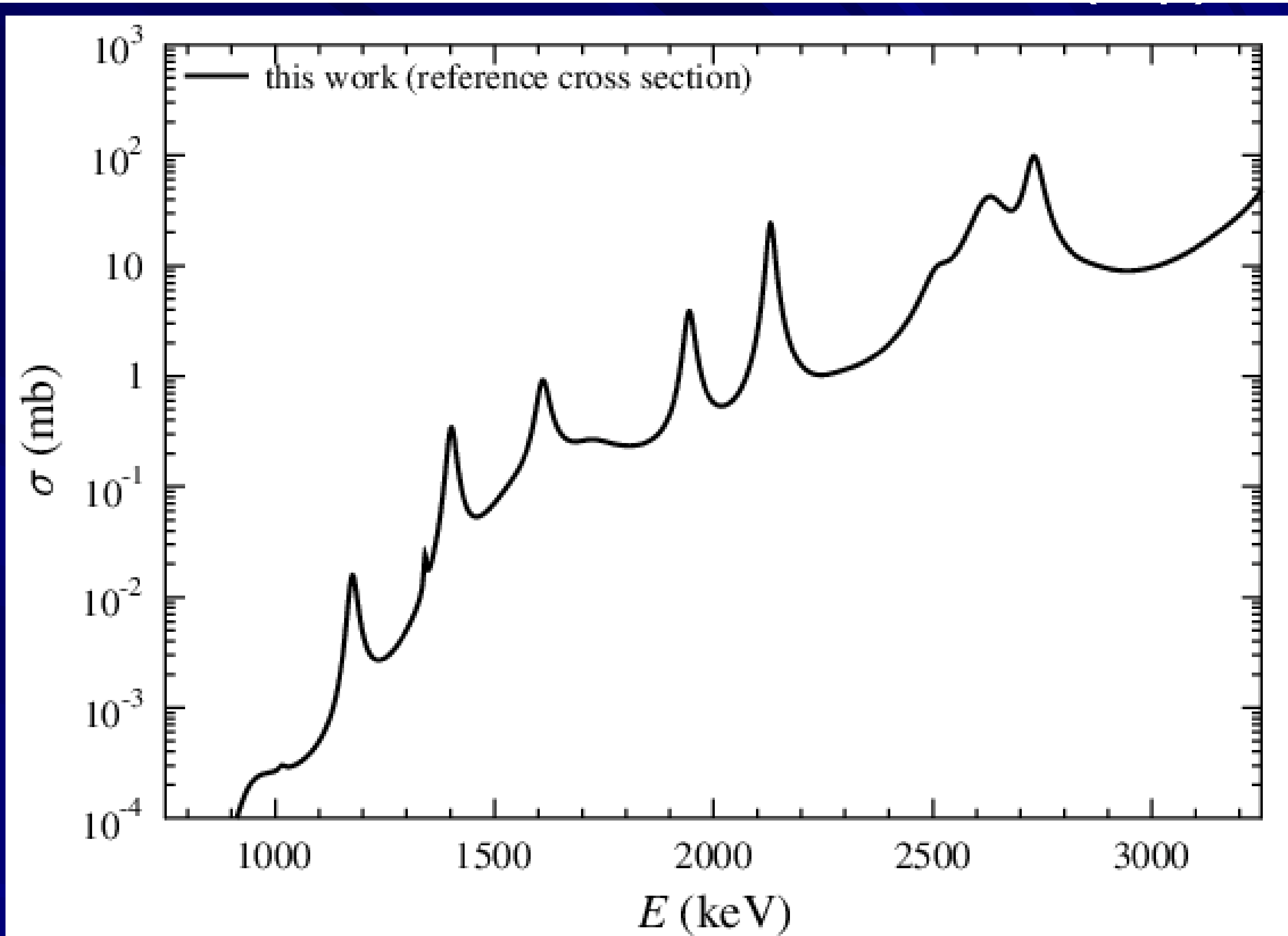
- resonance energy *E*
- angular momentum *L* (spin and parity J^π)
- (dimensionless) reduced width θ_α^2 (and radius *R*)

Definitions: reference cross section and reference rate

- ***Reference*** quantities σ_{ref} and $N_A < \sigma\, v >_{ref}$ are not the final ***recommended*** results!!!
  - useful for comparison of different results

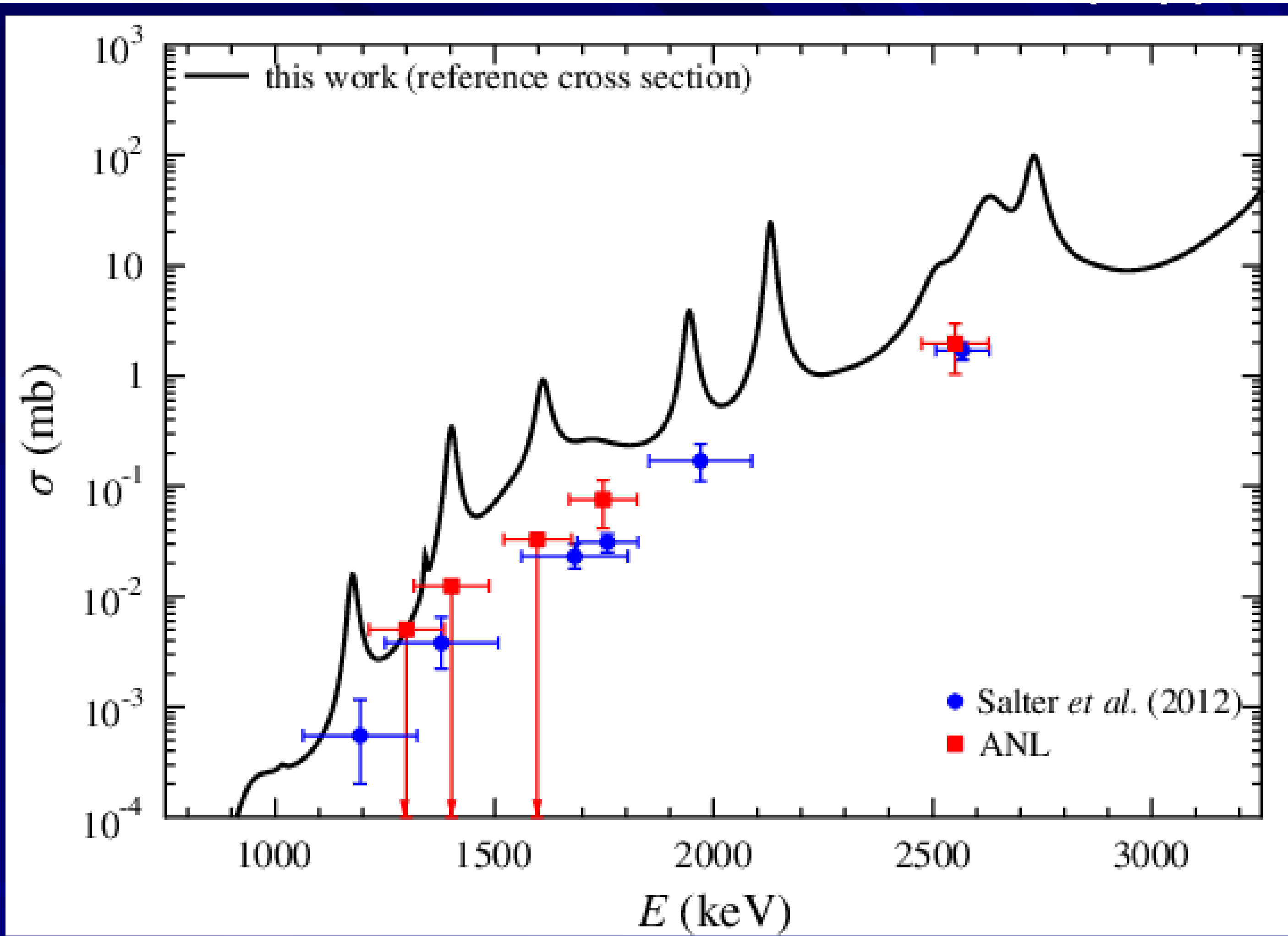
- Calculation of reference cross section σ_{ref} from:
  - resonance energies E from transfer (Matic 2009)
  - total widths Γ from transfer (Matic 2009)
  - spins and parities J^{π} from transfer (various sources)
  - resonance strengths $\omega\gamma$ from calculated Γ_{α}
    reduced widths θ_{α}^2 from α-transfer on ^{18}O mirror (adopted from Matic 2009)

The reference cross section for $^{18}Ne(\alpha,p)^{21}Na$



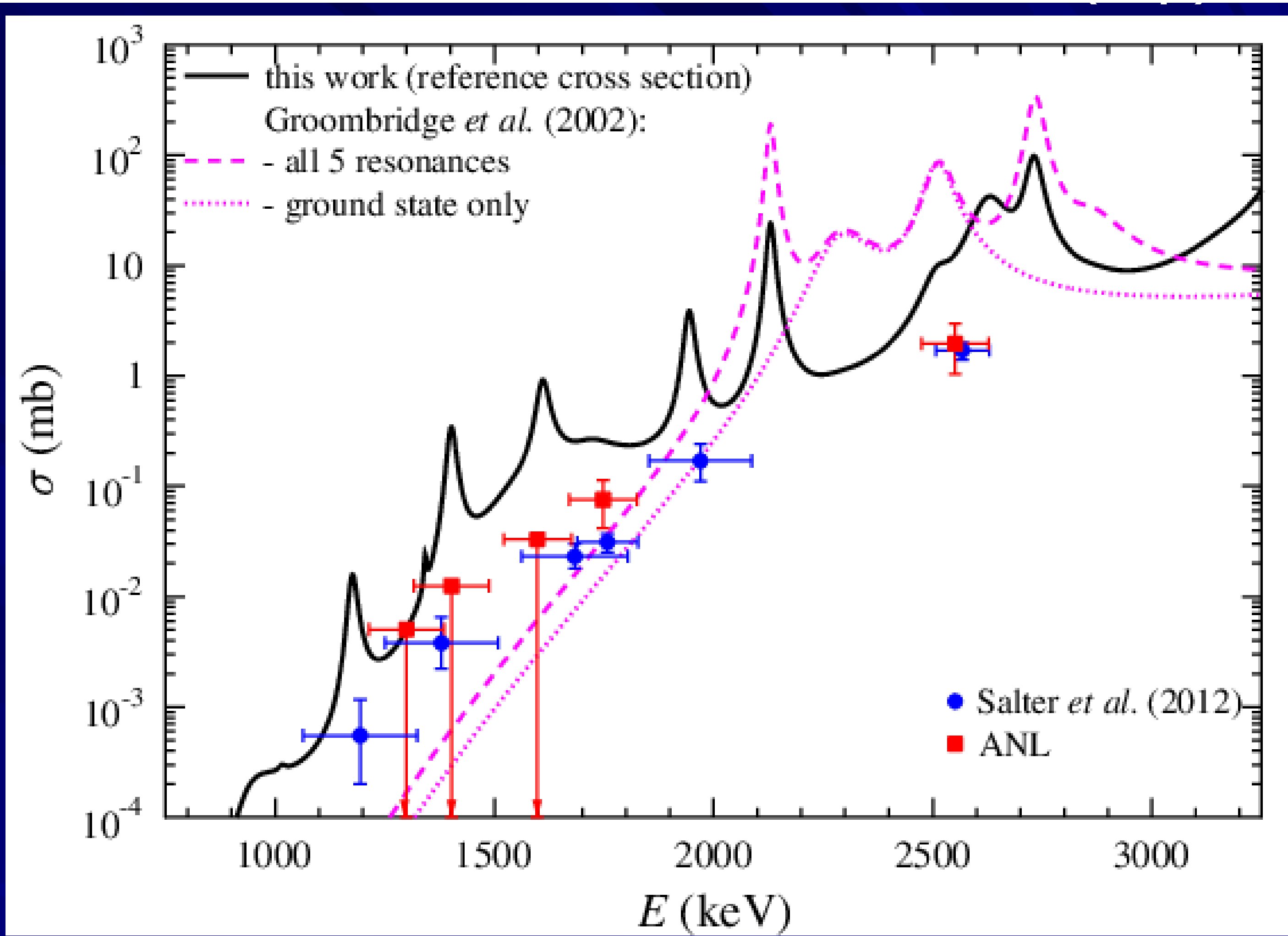
- Reference σ_{ref}: Mohr/Matic, Phys. Rev. C **87**, 035801 (2013) not yet the final recommended result!

The reference cross section for $^{18}Ne(\alpha,p)^{21}Na$



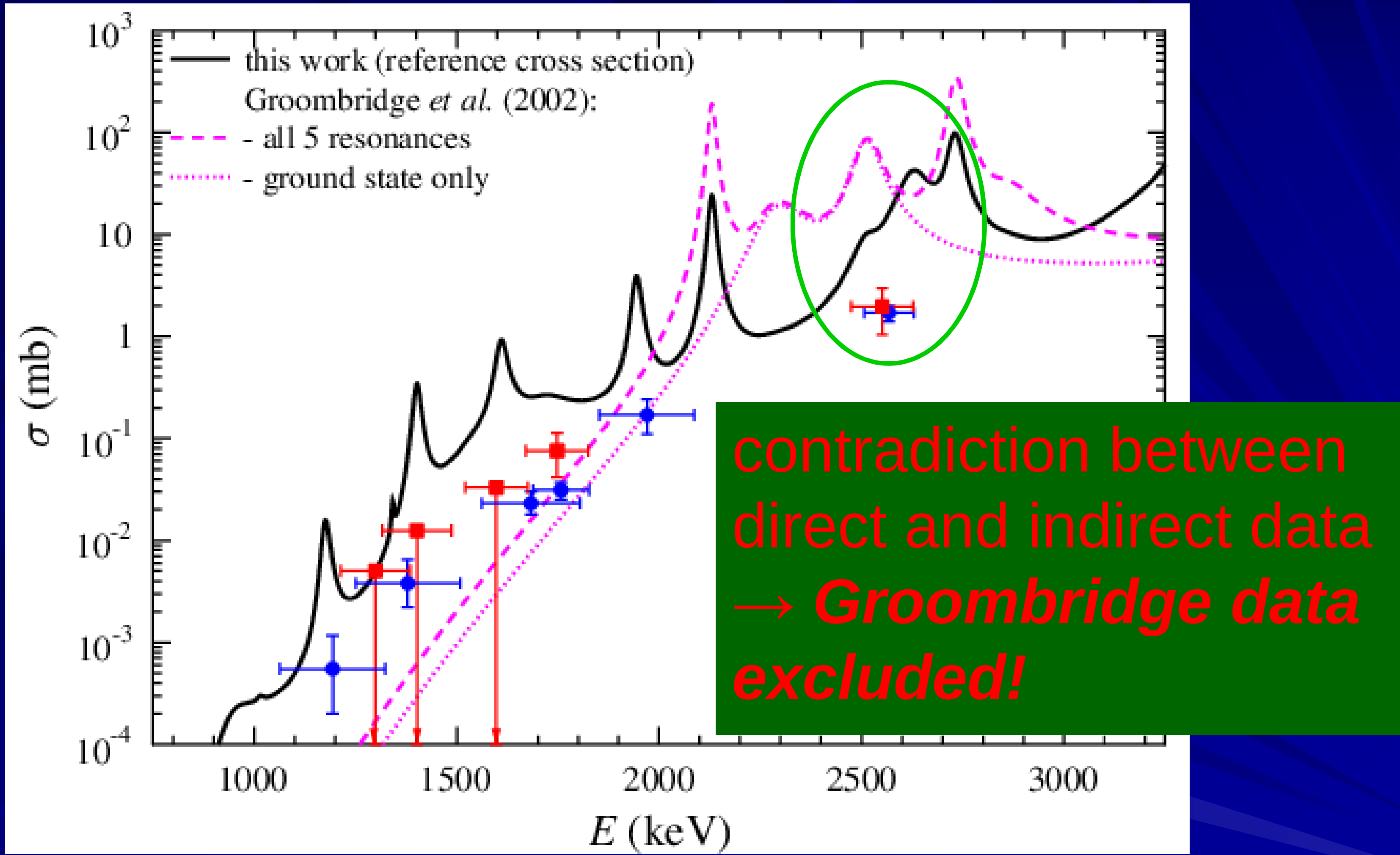
- compared to experimental reverse reaction data (ground state contribution only!)

The reference cross section for $^{18}Ne(\alpha,p)^{21}Na$



- and compared to the direct data by Groombridge (resonances with dominating ground state only)

The reference cross section for $^{18}Ne(\alpha,p)^{21}Na$



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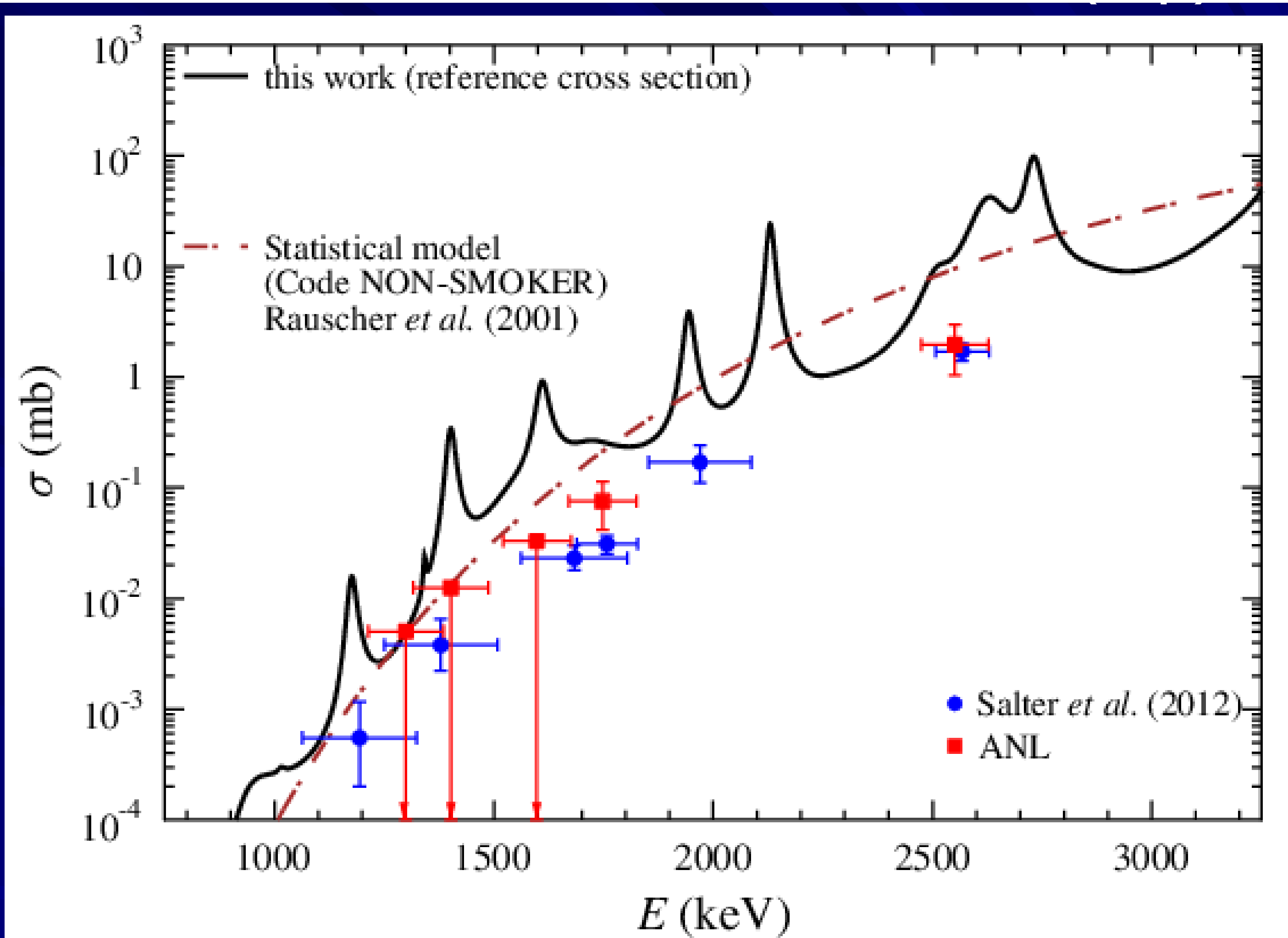
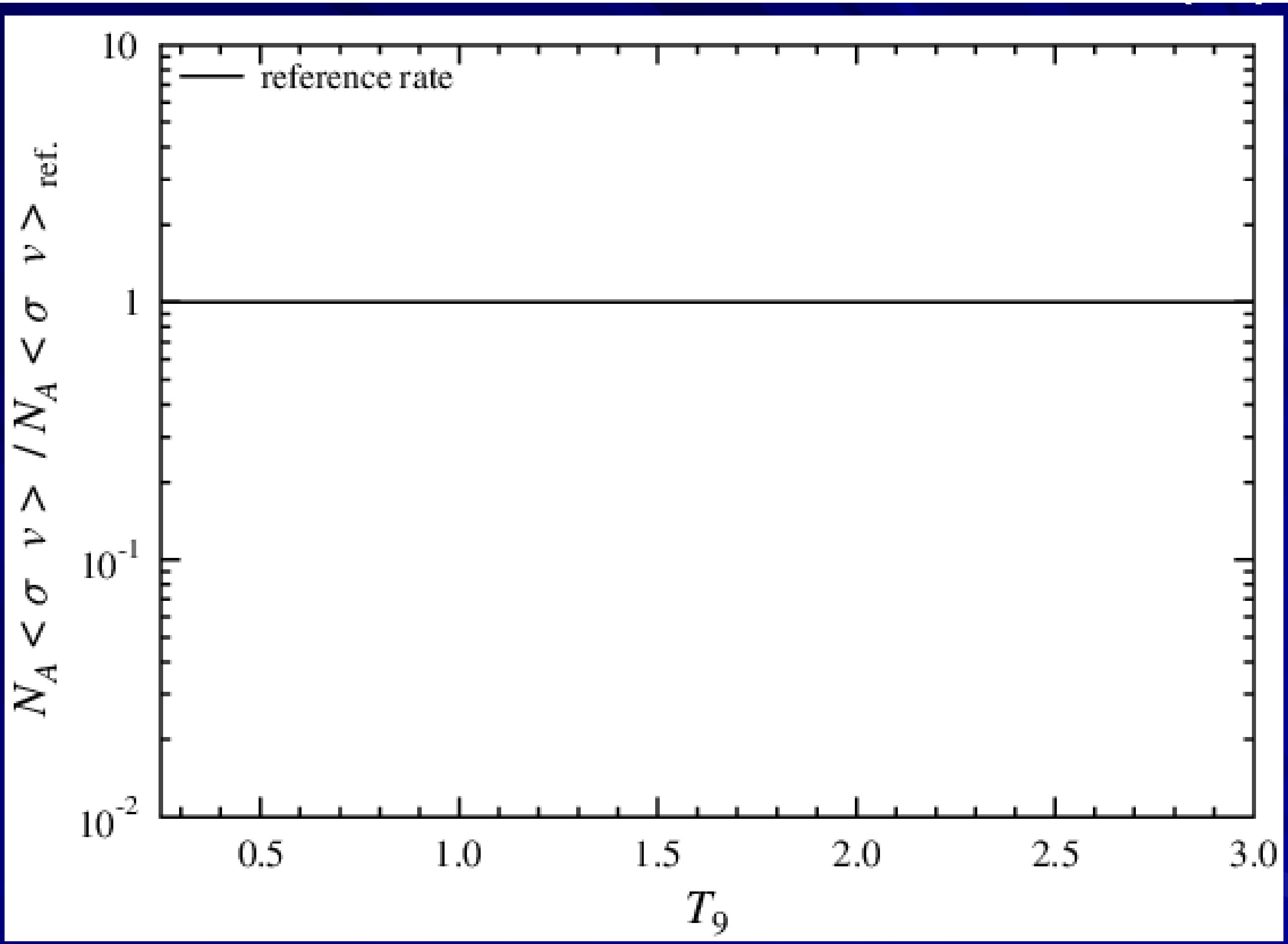
The reference cross section for $^{18}Ne(\alpha,p)^{21}Na$



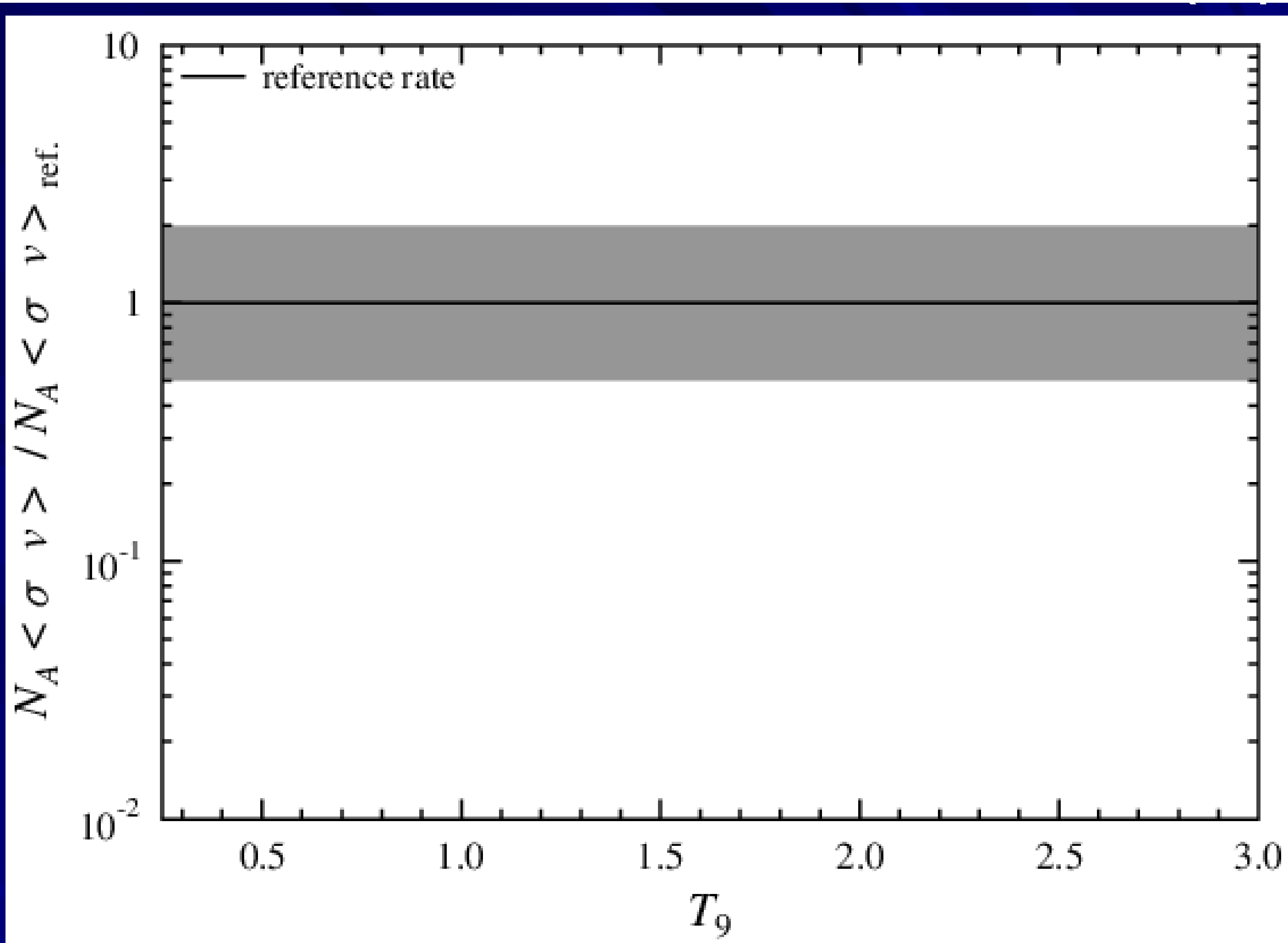
- compared to a statistical model calculation: cannot predict the individual resonances

Reference and recommended rate for $^{18}Ne(\alpha,p)^{21}Na$



- Rate $N_A < \sigma\ v >$ depends sensitively on temperature
  $\rightarrow$ rate normalized to reference rate $N_A < \sigma\ v >_{ref}$

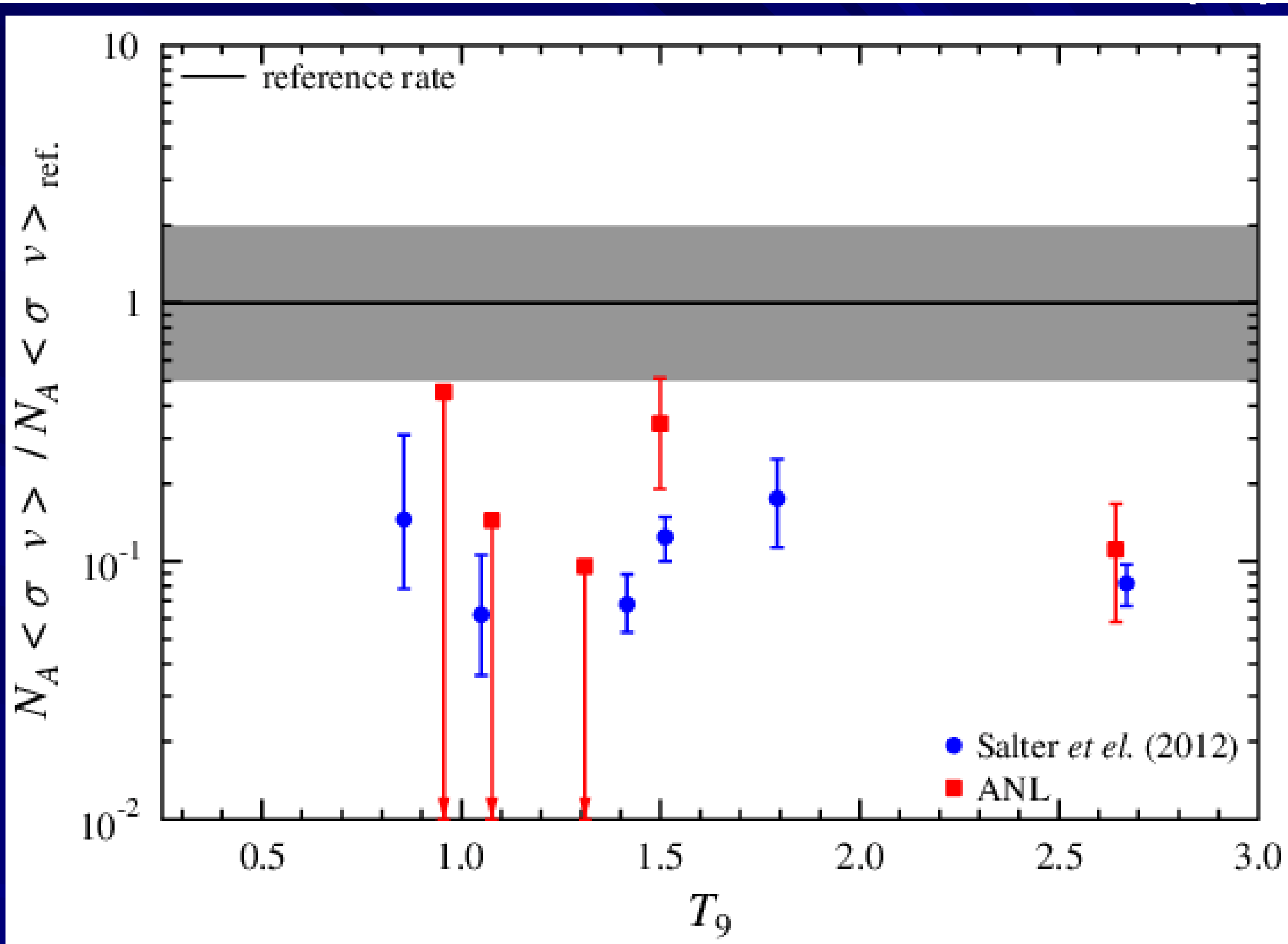
Reference and recommended rate for $^{18}Ne(\alpha,p)^{21}Na$



- Estimated uncertainty of reference: approx. factor 2

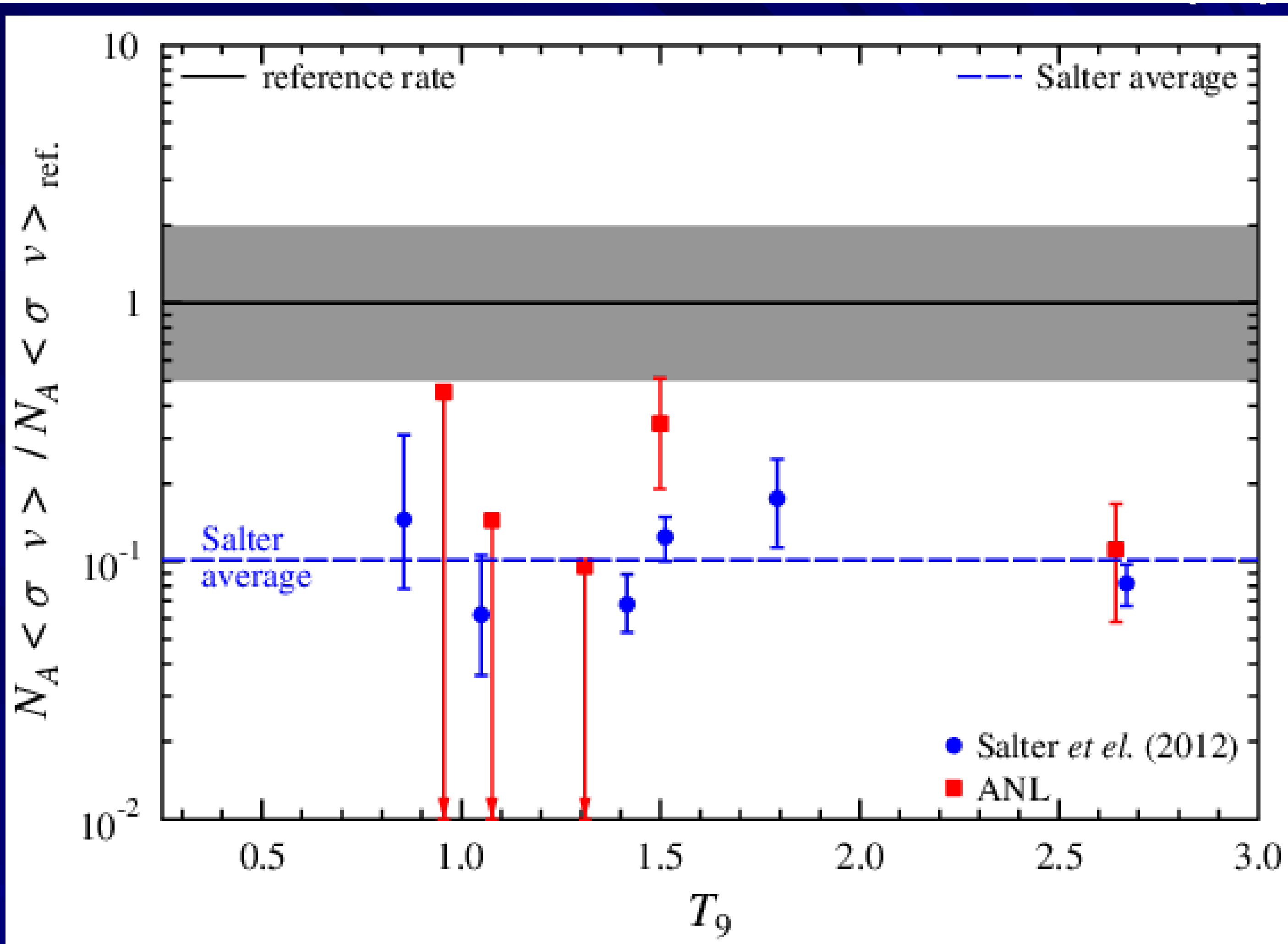
$0.5\ x\ N_A < \sigma\ v >_{ref} \leq N_A < \sigma\ v > \leq 2.0\ x\ N_A < \sigma\ v >_{ref}$

Reference and recommended rate for $^{18}Ne(\alpha,p)^{21}Na$



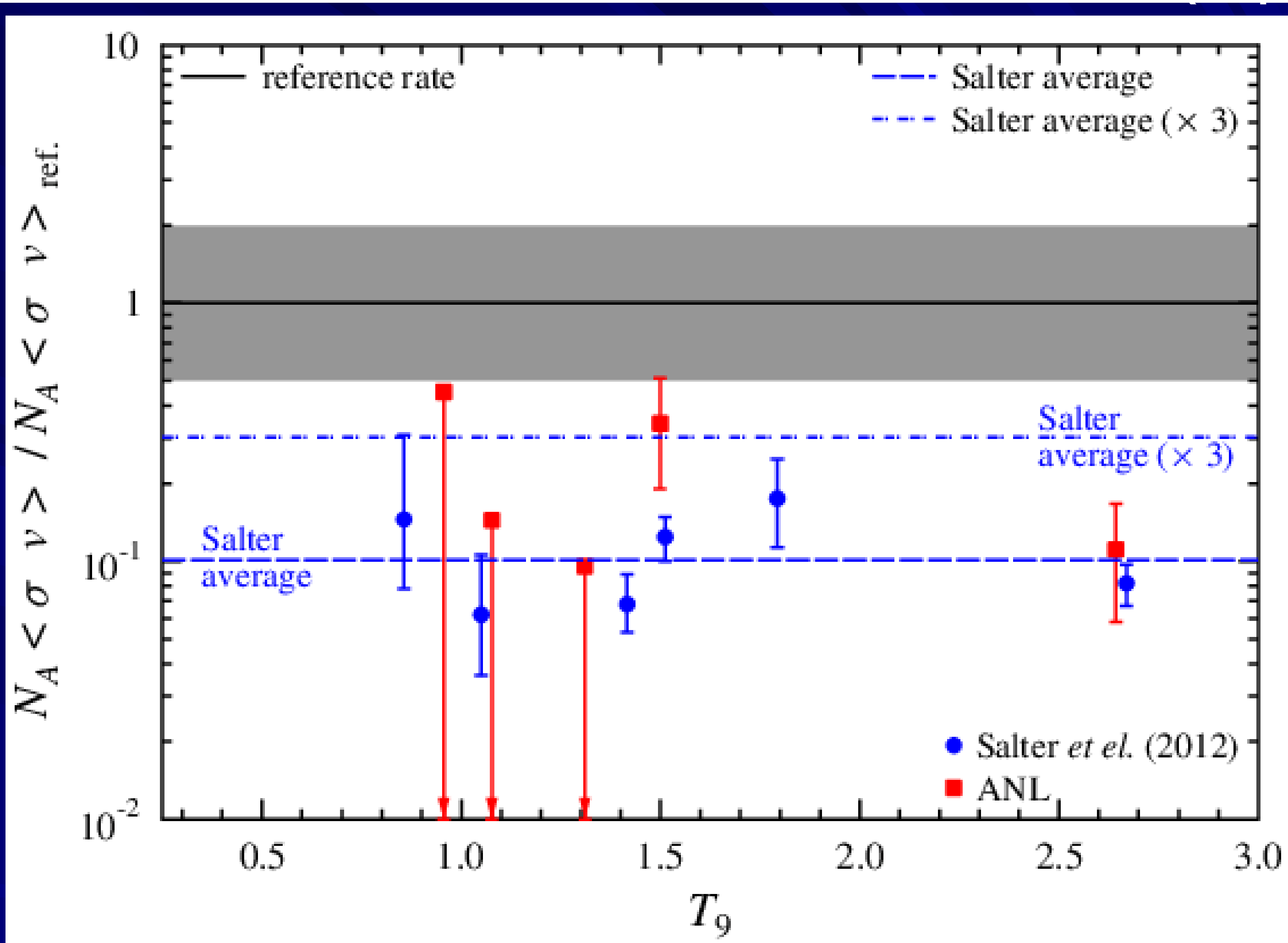
- Comparison to experimental reverse reaction data (ground state contribution only!)

Reference and recommended rate for $^{18}Ne(\alpha,p)^{21}Na$



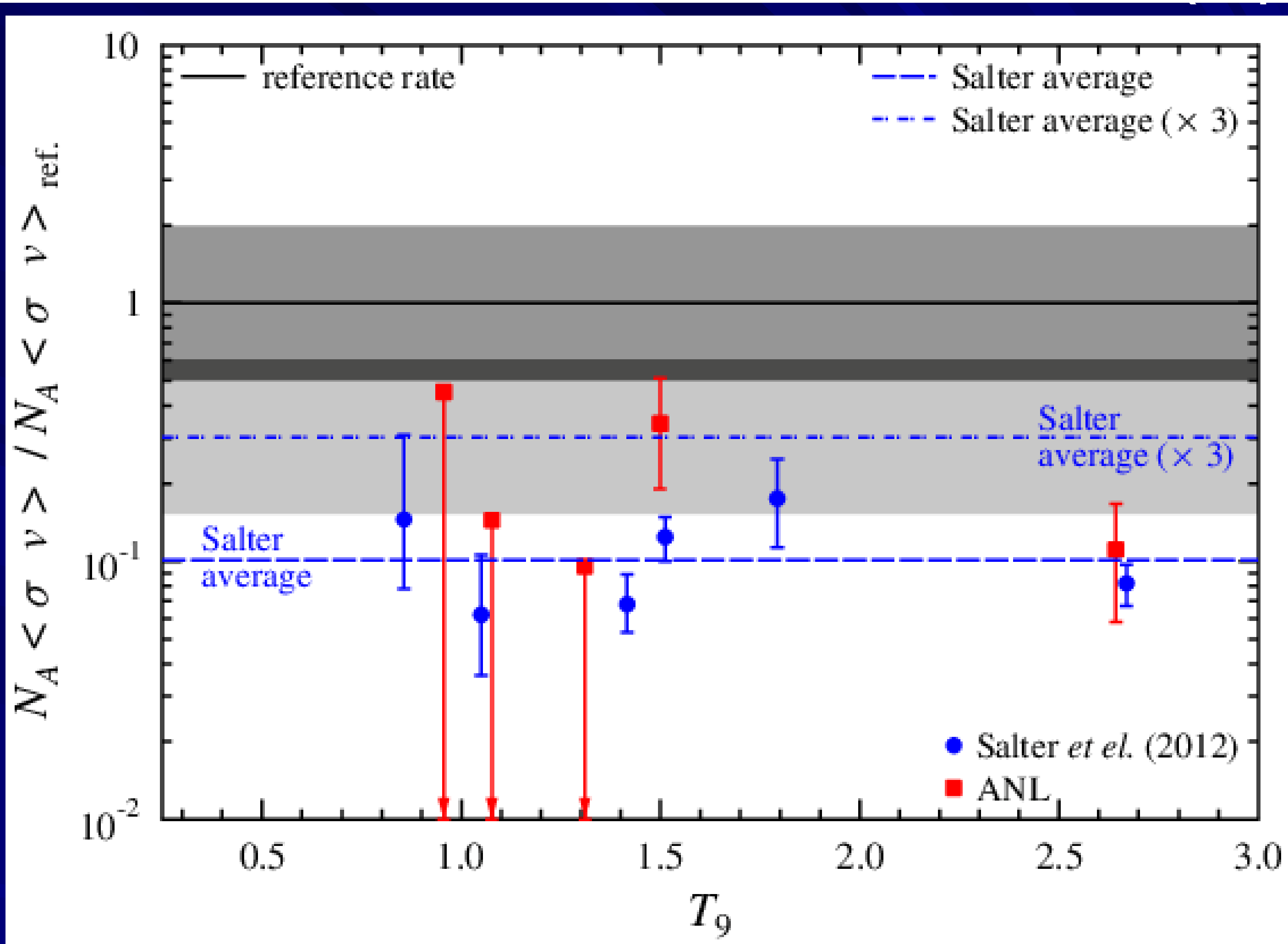
- Average of Salter reverse reaction data (g.s. only): $N_A < \sigma\ v > \approx 0.1\ x\ N_A < \sigma\ v >_{ref}$

Reference and recommended rate for $^{18}Ne(\alpha,p)^{21}Na$



- Salter data (corrected for excited state contributions):
$N_A < \sigma\, v > \approx 0.3 \times N_A < \sigma\, v >_{ref}$

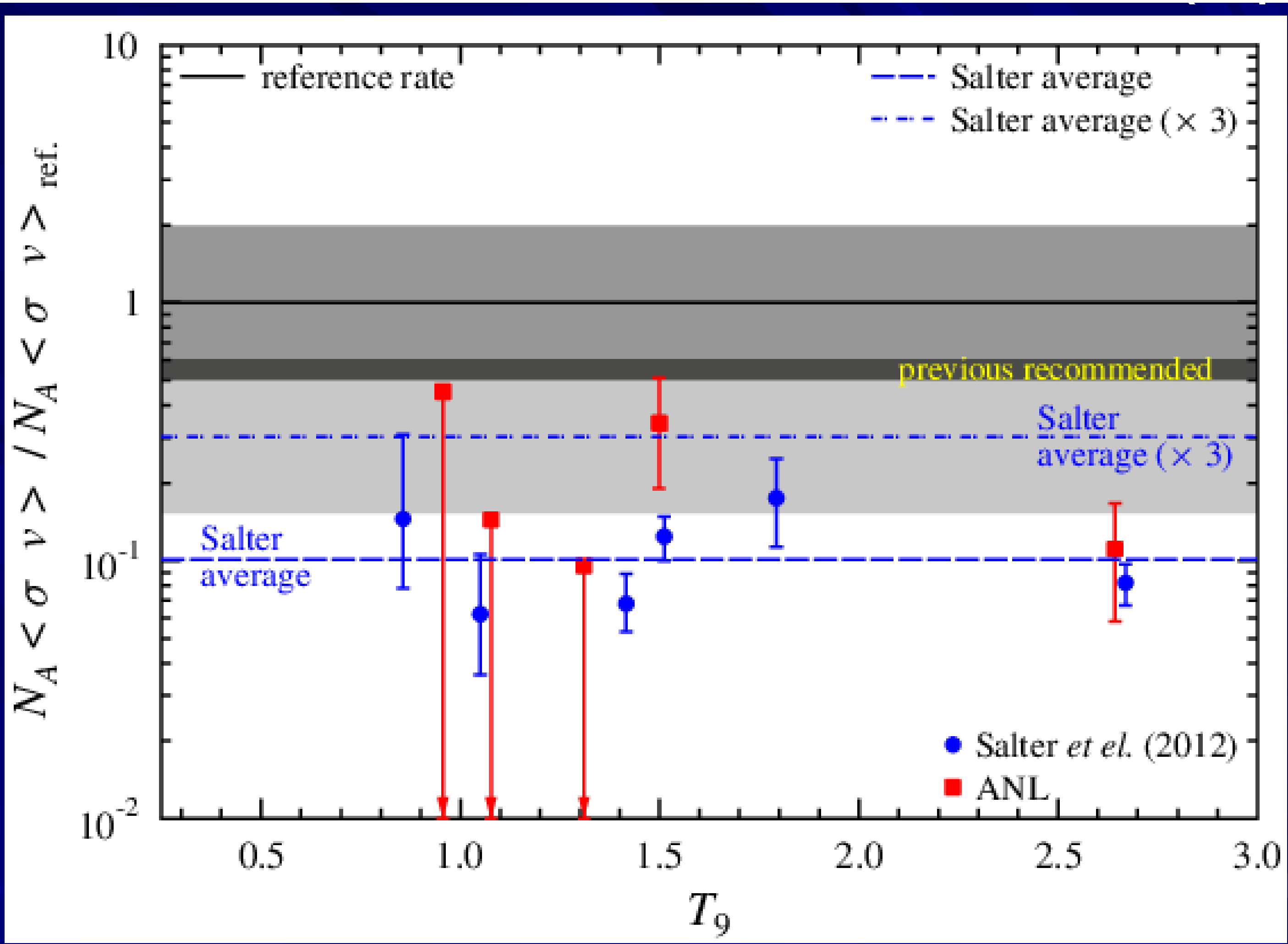
Reference and recommended rate for $^{18}Ne(\alpha,p)^{21}Na$



- estimated uncertainty of corrected data: approx. fact 2

$0.15 \; x \; N_A < \sigma \; v >_{ref} \leq N_A < \sigma \; v > \leq 0.6 \; x \; N_A < \sigma \; v >_{ref}$

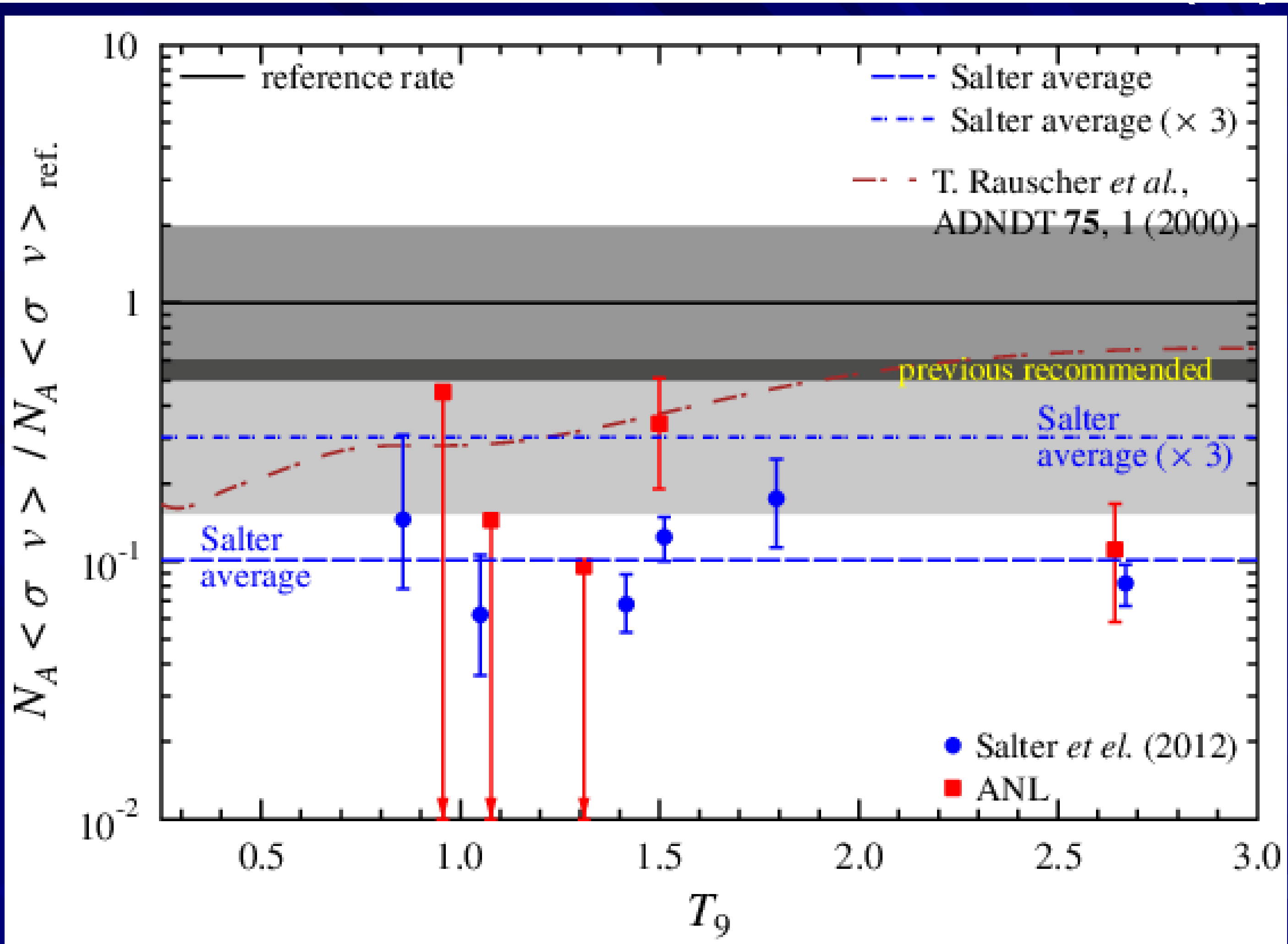
Reference and recommended rate for $^{18}Ne(\alpha,p)^{21}Na$



- previous recommended rate (Mohr/Matic 2013):

$0.50 \times N_A < \sigma\ v >_{ref} \leq N_A < \sigma\ v > \leq 0.60 \times N_A < \sigma\ v >_{ref}$

Reference and recommended rate for $^{18}Ne(\alpha,p)^{21}Na$



- Comparison to statistical model (NON-SMOKER): recommended $N_A < \sigma\ v >$ reproduced within factor 2

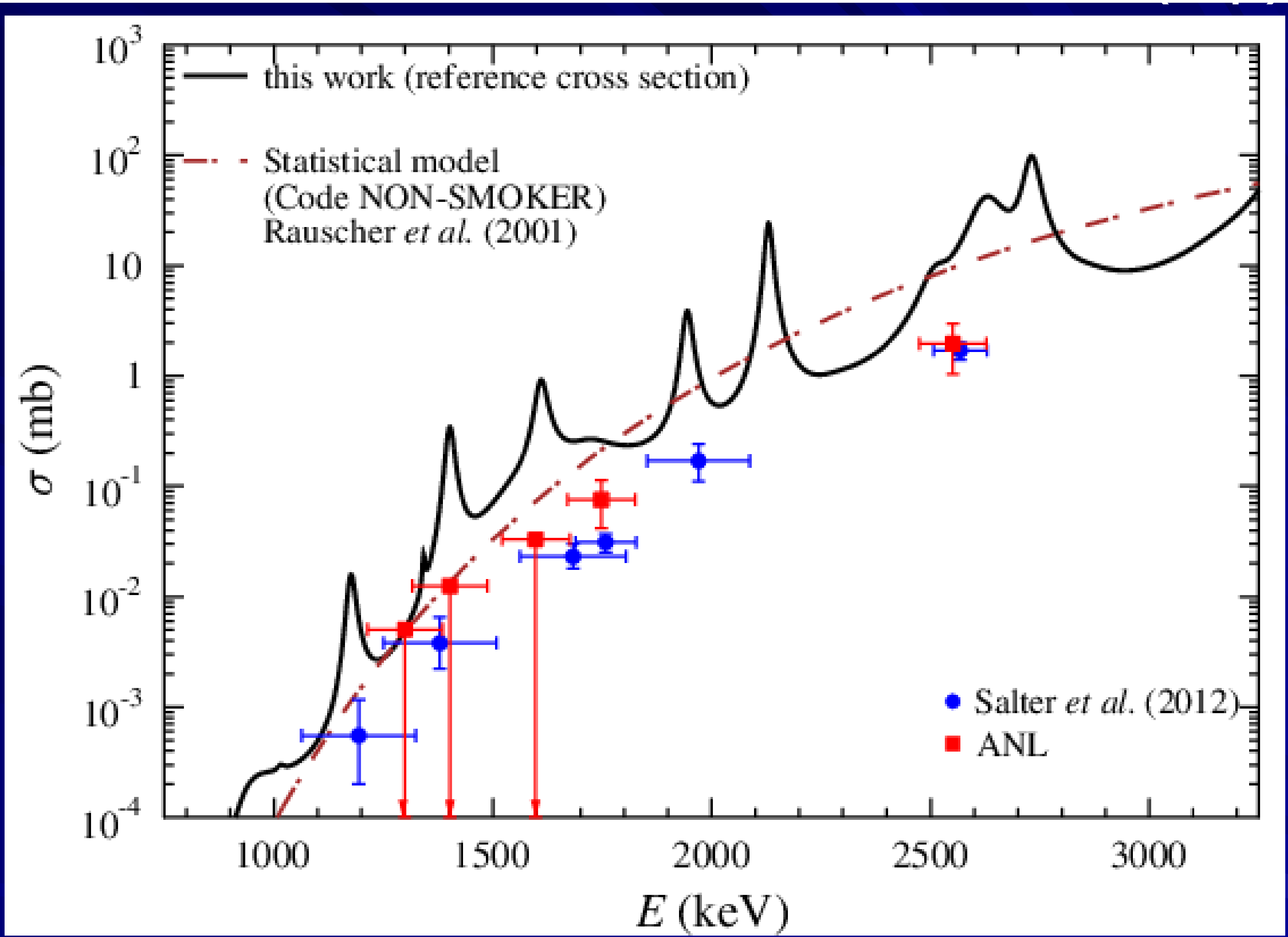
Progress in the last year (since Mohr/Matic 2013)

- J^{π} from ^{21}Na(p,p)^{21}Na resonant elastic scattering:
  Zhang *et al*., Phys. Rev. C **89**, 015804 (2014)
  - most tentative assignments of Matic *et al.* confirmed
  - 243 keV resonance excluded (unnatural parity)
  - rate practically identical to Mohr/Matic 2013 for relevant temperature range $1.0 \leq T_9 \leq 2.0$

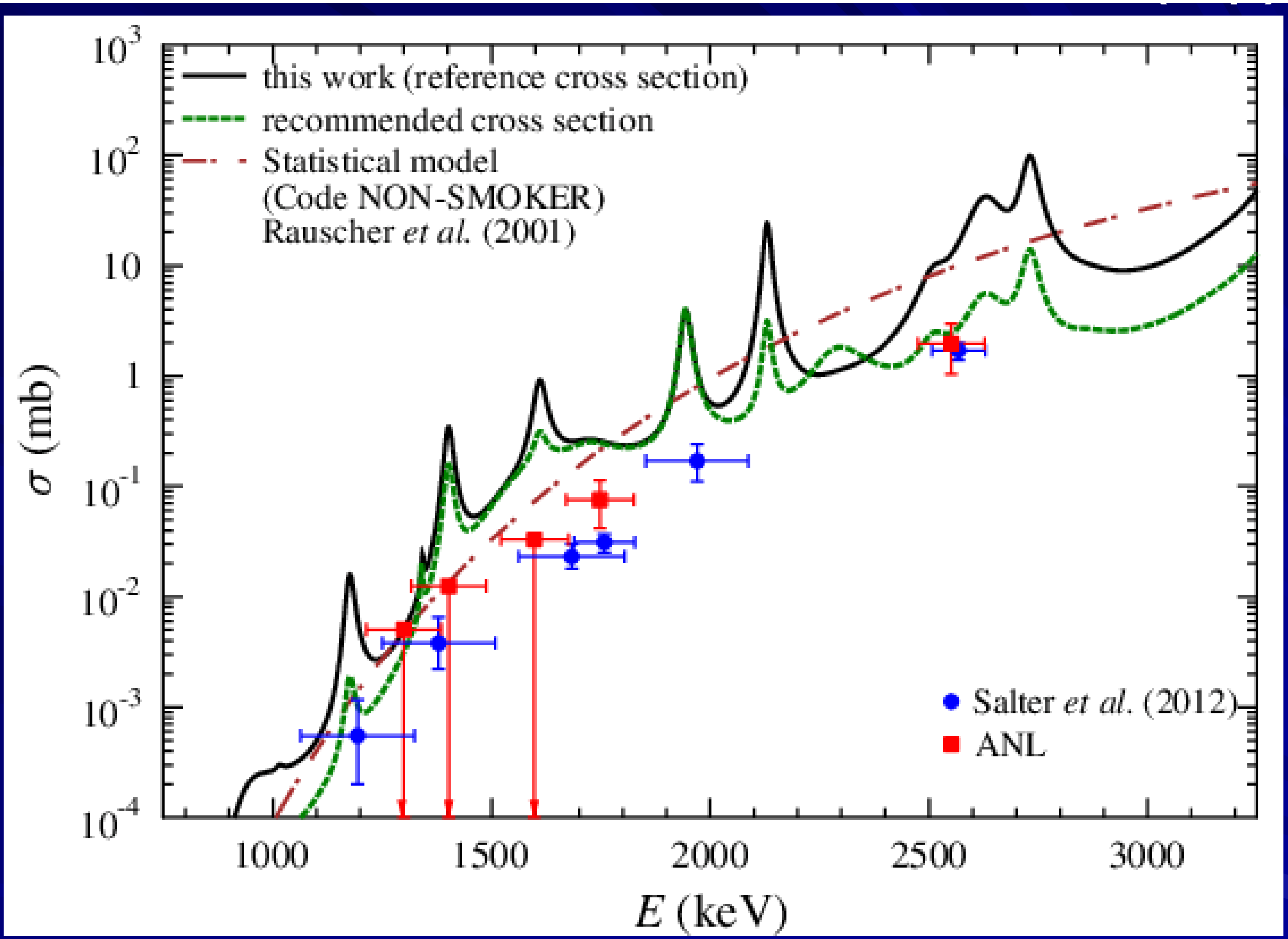
Progress in the last year (since Mohr/Matic 2013)

- Idea: apply the Monte-Carlo method for an improved determination of the rate factor and its uncertainties
  Longland *et al*., Nucl. Phys. **A841**, 1 (2010)
- Monte-Carlo sampling provides a consistent treatment of uncertainties in **all** relevant ingredients:
  - resonance energies *E*, spins and parities J^{π},
    partial widths Γ_{α}, resonance strengths ωγ
- Re-determination of all 33 resonance strengths ωγ
  - minor inconsistencies for θ_{α}^{2} in Matic 2009
  - reduced widths from Porter-Thomas distribution for
    resonances without spectroscopic information
  Pogrebnyak *et al*., Phys. Rev. C **88**, 015808 (2013)

The recommended cross section for $^{18}Ne(\alpha,p)^{21}Na$

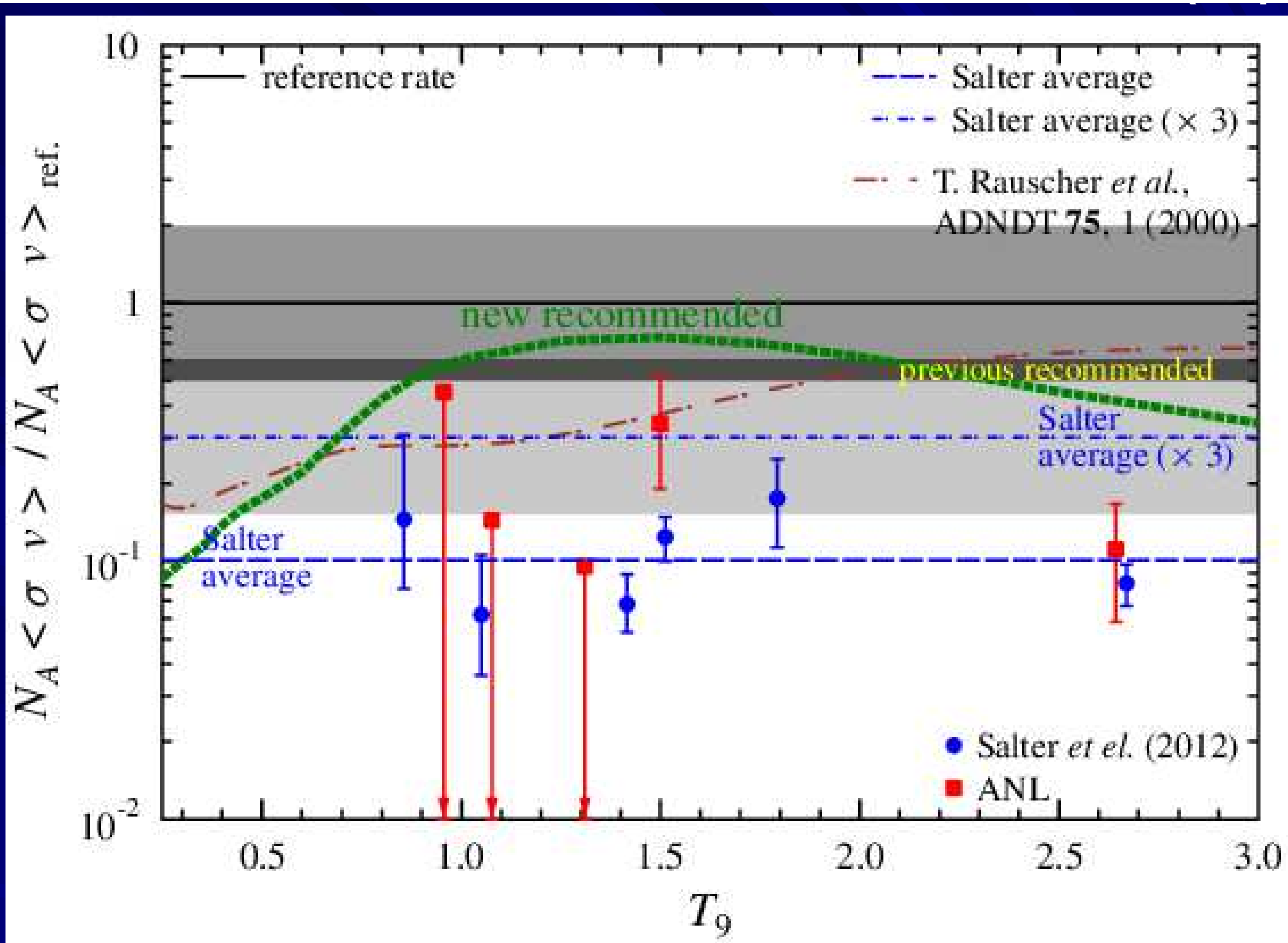


The recommended cross section for $^{18}Ne(\alpha,p)^{21}Na$



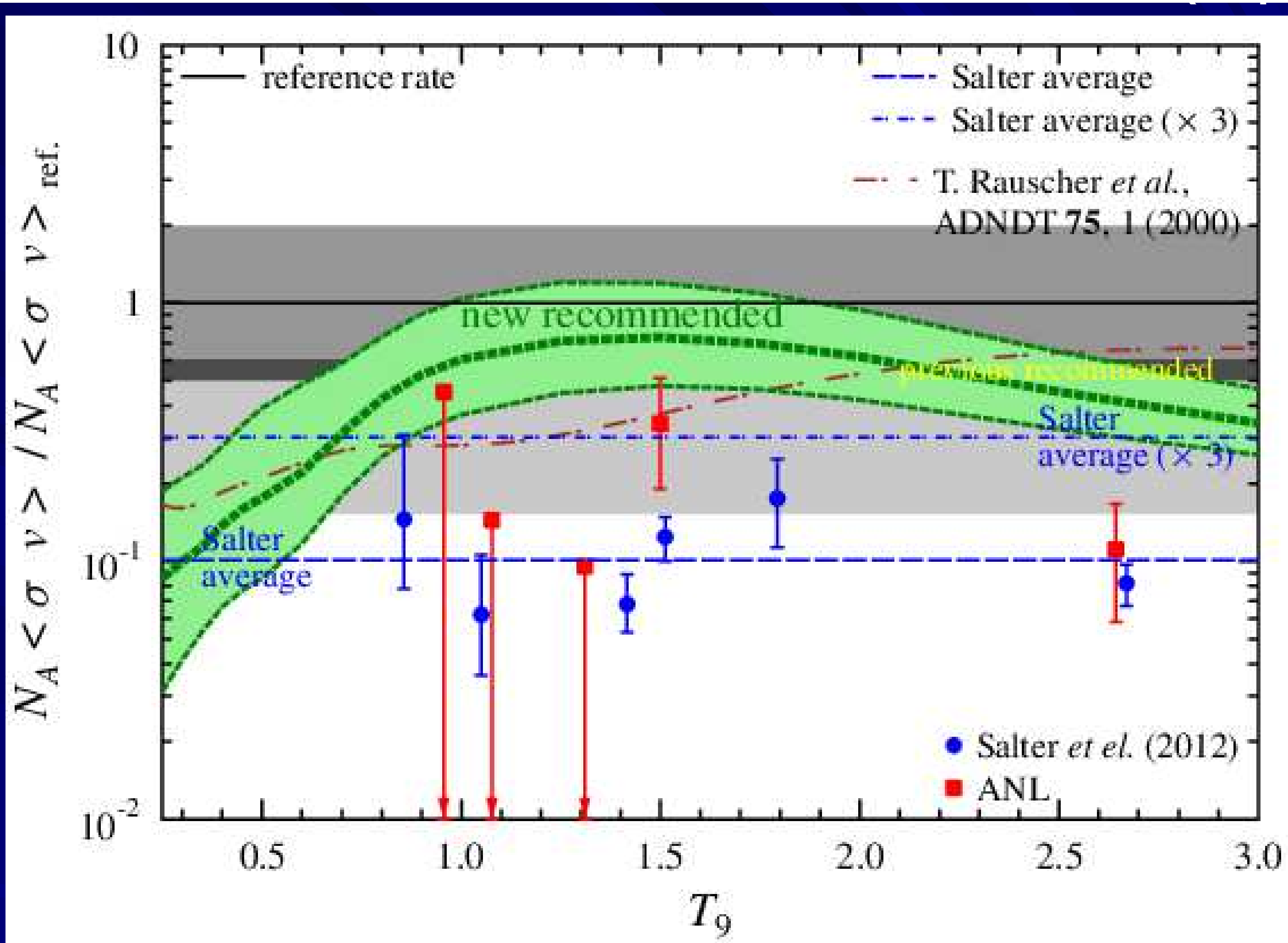
- Recommended cross section from the re-determined resonance strengths …

Reference and recommended rate for $^{18}Ne(\alpha,p)^{21}Na$



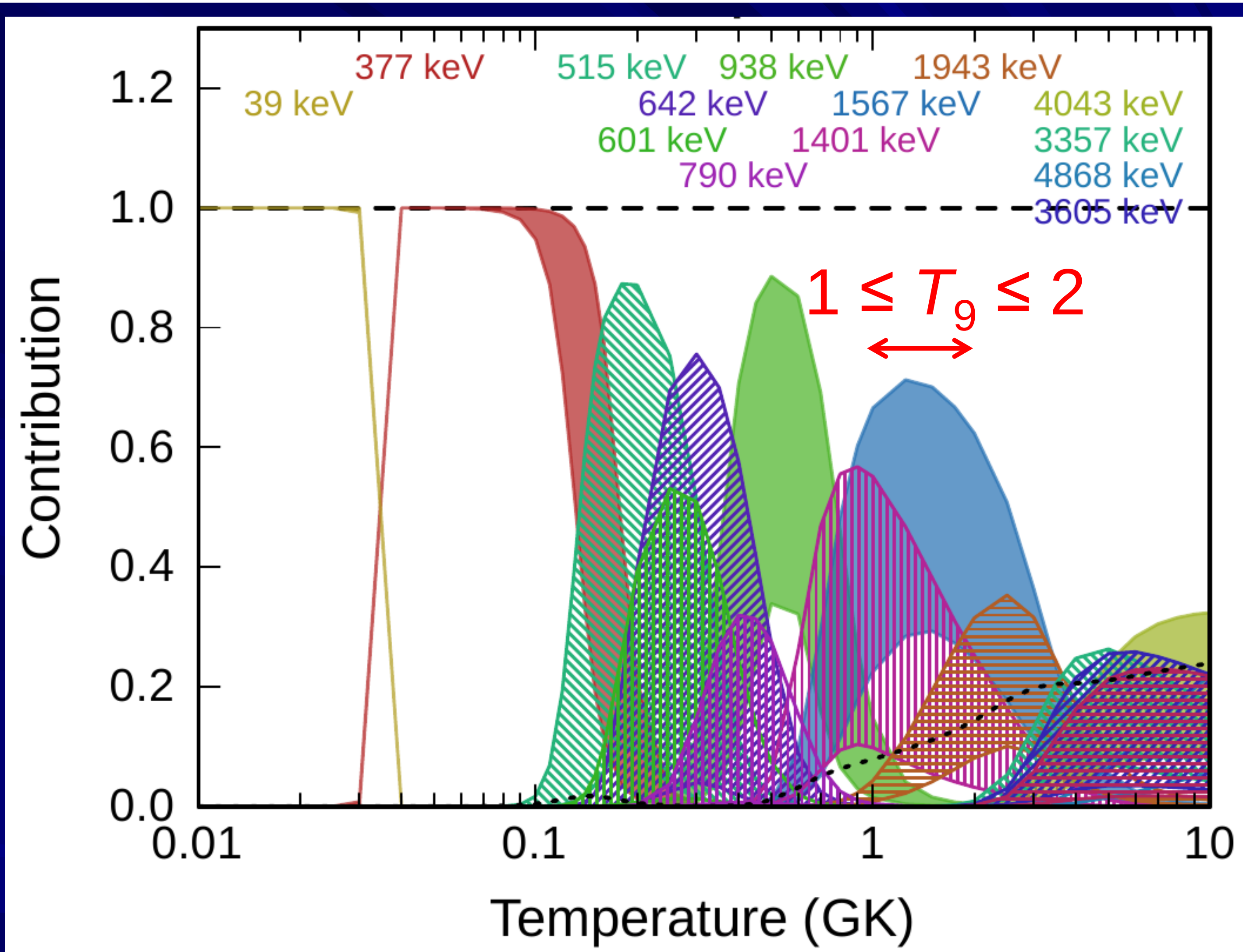
- New recommended $N_A < \sigma\ v >$ from MC sampling remains close to previous recommendation ($1 \leq T_9 \leq 2$)

Reference and recommended rate for $^{18}Ne(\alpha,p)^{21}Na$



- Uncertainties of new recommended $N_A < \sigma\, v >$ from MC sampling better constrained than before

Contributions of individual resonances to the rate



- Dominating resonances: 1^- (1401 keV), 0^+ (1567 keV)

Properties of dominating resonances

- 1^-, E = 1401 keV, E_x = 9542 keV:
  - total width: $\Gamma \leq$ 23 keV
  - partial width Γ_α = 1.91 eV
  - reduced width θ_α^2 = 0.12 from mirror $^{18}O(^6Li,d)^{22}Ne$
- 0^+, E = 1567 keV, E_x = 9709 keV:
  - total width: Γ = 268 ± 48 keV: ***difficult to measure!***
  - partial width Γ_α = 65.0 eV
  - reduced width θ_α^2 = 0.458 (huge value!)

    from mirror $^{18}O(\alpha,\gamma)^{22}Ne$ and $^{18}O(^6Li,d)^{22}Ne$

    also theoretically expected

    Descouvemont, Phys. Rev. C **38**, 2397 (1988)

    Kimura, Phys. Rev. C **75**, 034312 (2007)

    Levai, Phys. Rev. C **88**, 014328 (2013)

Summary

- Available experimental information for ^{18}Ne(α,p)^{21}Na: check of compatibility excludes Groombridge data
- Re-determination of all 33 resonance strengths ωγ from improved estimates of reduced widths θ_α^2
- Monte-Carlo sampling of available parameter space for realistic estimate of uncertainties: ≤ factor 2
- New recommended $N_A < \sigma v >$ close to previous recommendation → Mohr/Matic, PRC **87**, 035801 (2013)
- Break-out from hot CNO to rp-process at $T_9 \approx 0.6$ for typical parameters in X-ray bursters (density $\rho = 10^6$ g/cm^3, α mass fraction $Y_\alpha = 0.27$)

Outlook

- Rates of (α,p) reactions for targets with $20 < A < 50$ may be much more uncertain than estimated before
- Typically (at least for heavy targets down to $A \approx 60$): statistical model **over**estimates σ(α,*X*) – however:

- **reasonable** prediction for ^{18}Ne(α,p)^{21}Na

- **dramatic under**prediction for ^{23}Na(α,p)^{26}Mg

  Almarez-Calderon *et al.*, Phys. Rev. Lett. **112**, 152701 (2014)

- **significant under**prediction for ^{33}S(α,p)^{36}Cl

  Bowers *et al.*, Phys. Rev. C **88**, 065802 (2013)
  Mohr, Phys. Rev. C **89**, 058801 (2014)

- **minor under**prediction for ^{44}Ti(α,p)^{47}V

  Margerin *et al.*, Phys. Lett. B **731**, 358 (2014)

Thank you very much for your attention!

Further resonances

- 1^-, E = 938 keV, E_x = 9080 keV:
  - total width: Γ = 114 ± 20 keV: difficult to measure!!!
  - partial width Γ_α = 1.75 meV
  - reduced width θ_α^2 = 0.064
    from mirror ^{18}O(^{6}Li,d)^{22}Ne
- 2^+, E = 1943 keV, E_x = 10085 keV:
  - total width: Γ = 26 ± 9 keV
  - partial width Γ_α = 46.8 eV
  - reduced width θ_α^2 = 0.134
    from mirror ^{18}O(α,γ)^{22}Ne

Stellar reaction rate factor (indirect approach 1)

- For simplicity: narrow resonance formalism:

$$N_A < \sigma v > = 1.54 \times 10^{11} \, (A_{\rm red} \, T_9)^{-3/2} \times \sum_i (\omega\gamma)_i \times \exp\left(-11.605 \, E_i/T_9\right)$$

- Resonance energies E_i:
  enter exponentially into rate factor $N_A < \sigma v >$
  $E_i = E_{x,i}(^{22}\mathrm{Mg}) - S_\alpha = E_{x,i}(^{22}\mathrm{Mg}) - 8.142$ MeV

- Resonance strengths $(\omega\gamma)_i$:
  enter linearly into rate factor $N_A < \sigma v >$

- In total: 33 resonances in the relevant energy range

An interesting cancellation effect

- well-established *s*-wave (L=0) α-strength in ^{22}Ne from ^{18}O(α,γ)^{22}Ne and ^{18}O(^{6}Li,d)^{22}Ne:
  0^+, E_x = 10066 keV, $\theta_\alpha^2 \approx 0.5$
- Mirror state in ^{22}Mg is located within Gamow window (for typical temperatures around $T_9 \approx 1 - 2$)
- Even if the precise mirror assignment is uncertain:
  $N_A < \sigma v >$ remains relatively stable!

  e.g.: mirror at higher energies:
  larger partial width $\Gamma_\alpha \rightarrow$ larger resonance strength ωγ almost exactly compensated by smaller exponential factor $\exp(-11.605E / T_9)$

Some Definitions

- (Forward) reaction: ^{18}Ne(α,p)^{21}Na
- (Forward) reaction, inverse kinematics: α(^{18}Ne,p)^{21}Na
  - exchange of projectile and target
    (for practical reasons)
  - probes exactly the same physics
- Reverse reaction: ^{21}Na(p,α)^{18}Ne
  - exchange of entrance and exit channels
  - probes different physics (but common: gs → gs)
- Reverse reaction, inverse kinematics: p(^{21}Na,α)^{18}Ne
  - exchange of projectile and target
    (for practical reasons)

Available experimental data (I): direct data

- ^{18}Ne(α,p)^{21}Na (in inverse kinematics):
  - Groombridge *et al*., Phys. Rev. C **66**, 055802 (2002)
  - Bradfield-Smith *et al*., Phys. Rev. C **59**, 3402 (1999)
- measured observables:
  - energy loss of ^{18}Ne beam
  - energy of ejectile protons
  - reconstruction of interaction vertex
- derived quantities:
  - (average) cross section $\langle\sigma\rangle$ 😐
  - decay branchings ☺
  - resonance strengths ωγ ☺ (if reliable…)
  - (resonance energies and J^{π}): only tentative 😐

Available data (II): reverse reaction data

- $^{21}Na(p,\alpha)^{18}Ne$ (in inverse kinematics):
  - Salter *et al*., Phys. Rev. Lett. **108**, 242701 (2012)
  - Sinha, Rehm, *et al*., ANL Annual Report 2005
- measured observables:
  - energy of ejectile α
- derived quantities:
  - (average) cross section <σ> 😐
    (ground state contribution only!) ☹

Available data (III): various transfer reactions

- ^{24}Mg(p,t)^{22}Mg:
  - Matic *et al*., Phys. Rev. C **80**, 055804 (2009)
    → best resolution (13 keV) ☺
  - Chae *et al*., Phys. Rev. C **79**, 055804 (2009)
- ^{12}C(^{16}O,^{6}He)^{22}Mg:
  - Chen *et al*., Phys. Rev. C **63**, 065807 (2001)
- ^{24}Mg(α,^{6}He)^{22}Mg:
  - Berg *et al*., Nucl. Phys. **A718**, 608 (2003)
- ^{24}Mg(^{3}He,^{6}He)^{22}Mg:
  - Caggiano *et al*., Phys. Rev. C **66**, 015804 (2002)
- derived quantities:
  - excitation energies E_x(^{22}Mg) ☺ and total widths Γ ☺
  - tentative J^π from (limited) angular distributions 😐

Available data (IV): further information

- ^{21}Na(p,p)^{21}Na resonant elastic scattering: determination of J^{π} in compound ^{22}Mg
  - He *et al*., Phys. Rev. C **88**, 012801(R) (2013)
- α-transfer in isospin mirror system ^{22}Ne:
  ^{18}O(α,γ)^{22}Ne:
  - Dababneh *et al*., Phys. Rev. C **68**, 025801 (2003); etc etc

  ^{18}O(^{6}Li,d)^{22}Ne:
  - Giesen *et al*., Nucl. Phys. **A567**, 146 (1994); etc etc

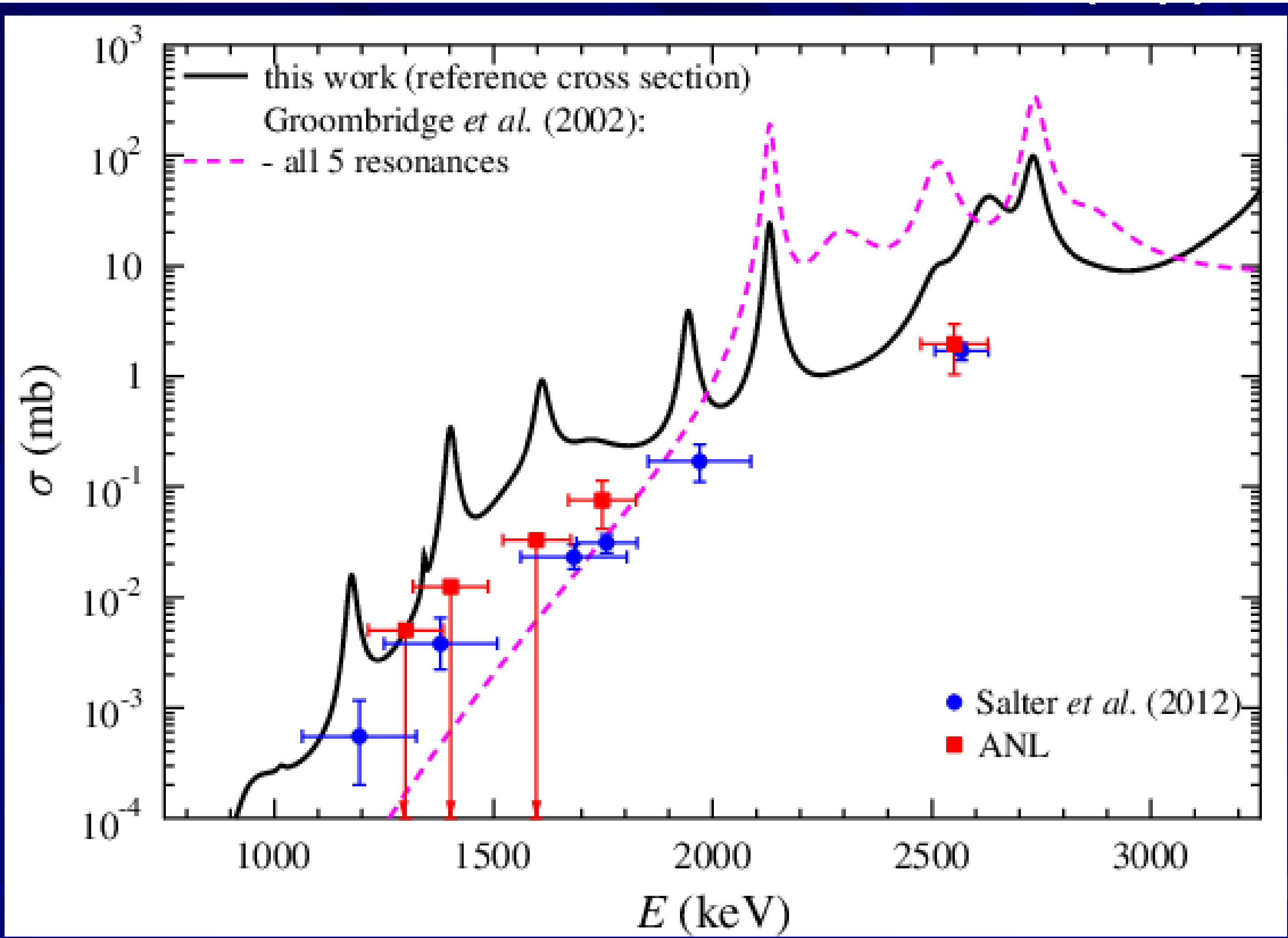
  - determination of width Γ_α, reduced width θ_α^2 in ^{22}Ne

  - assumption of similar wave functions in mirror nuclei
    i.e.: $\theta_\alpha^2(^{22}\mathrm{Ne}) = \theta_\alpha^2(^{22}\mathrm{Mg})$

Compatibility of available experimental data

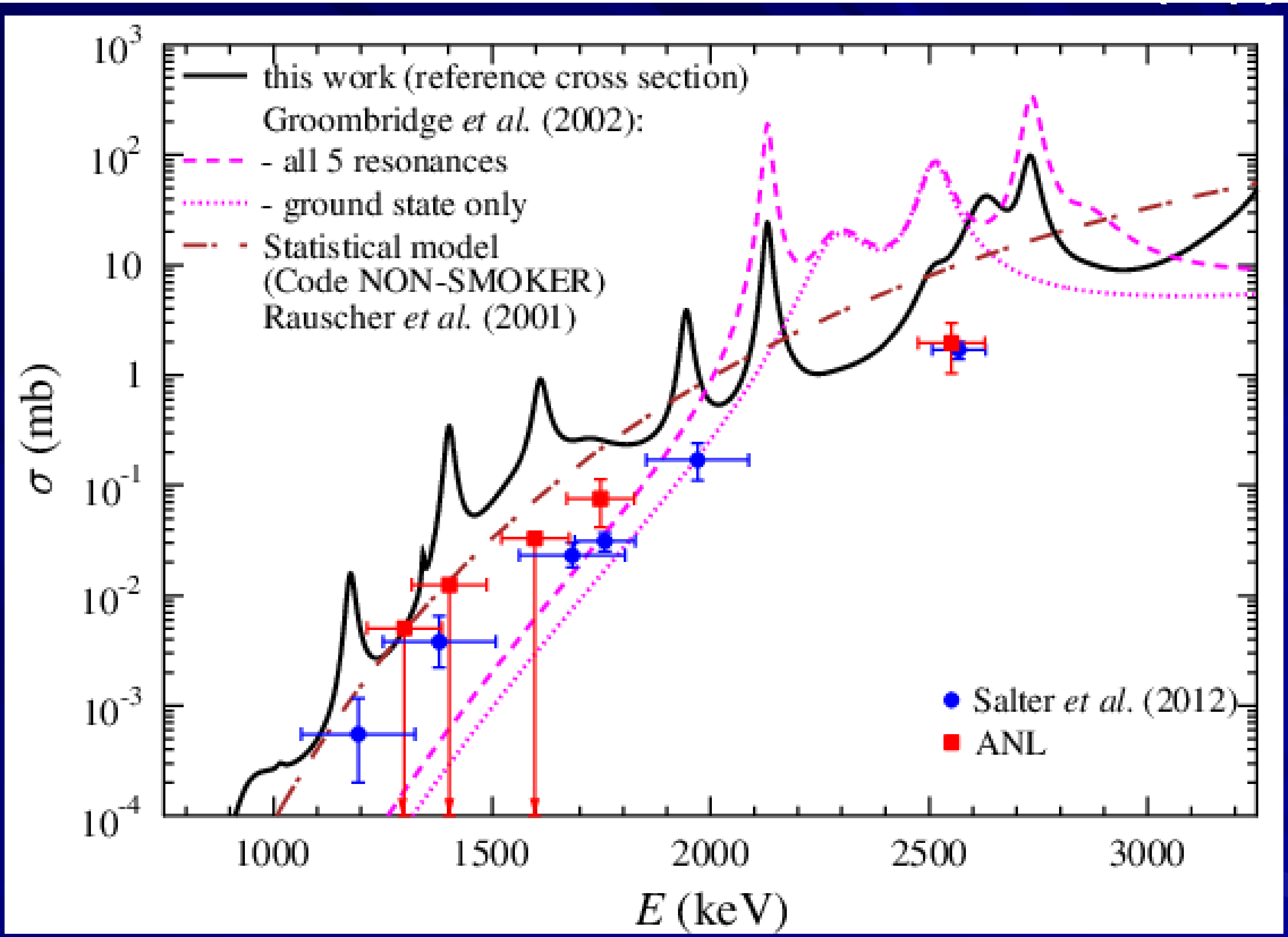
- Direct data vs. transfer data:
  - different observables → no contradictions
- Reverse reaction data vs. transfer data:
  - different observables → no contradictions
- Direct data vs. reverse reaction data:
  - common observable: σ(g.s.) → σ(g.s.)
  - direct data (Groombridge) are about a factor of 10 higher than reverse reaction data (Salter, ANL) ☹
- Additionally: direct data (Groombridge) are close to a theoretical upper limit (single-particle width)
- → Direct data (Groombridge) not used for $N_A < \sigma v >$

The reference cross section for $^{18}Ne(\alpha,p)^{21}Na$



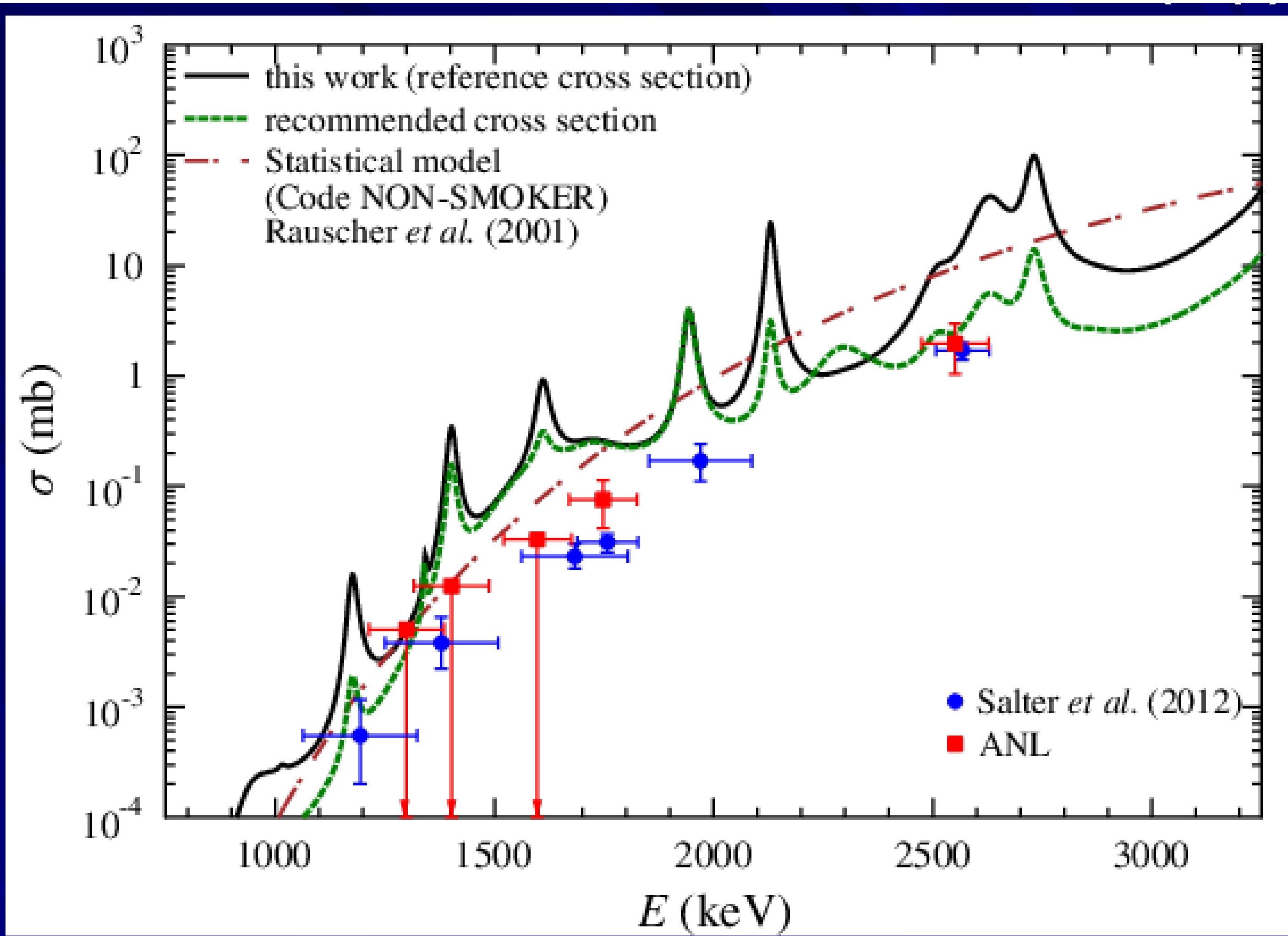
- and compared to the direct data by Groombridge (all 5 observed resonances)

The recommended cross section for $^{18}Ne(\alpha,p)^{21}Na$



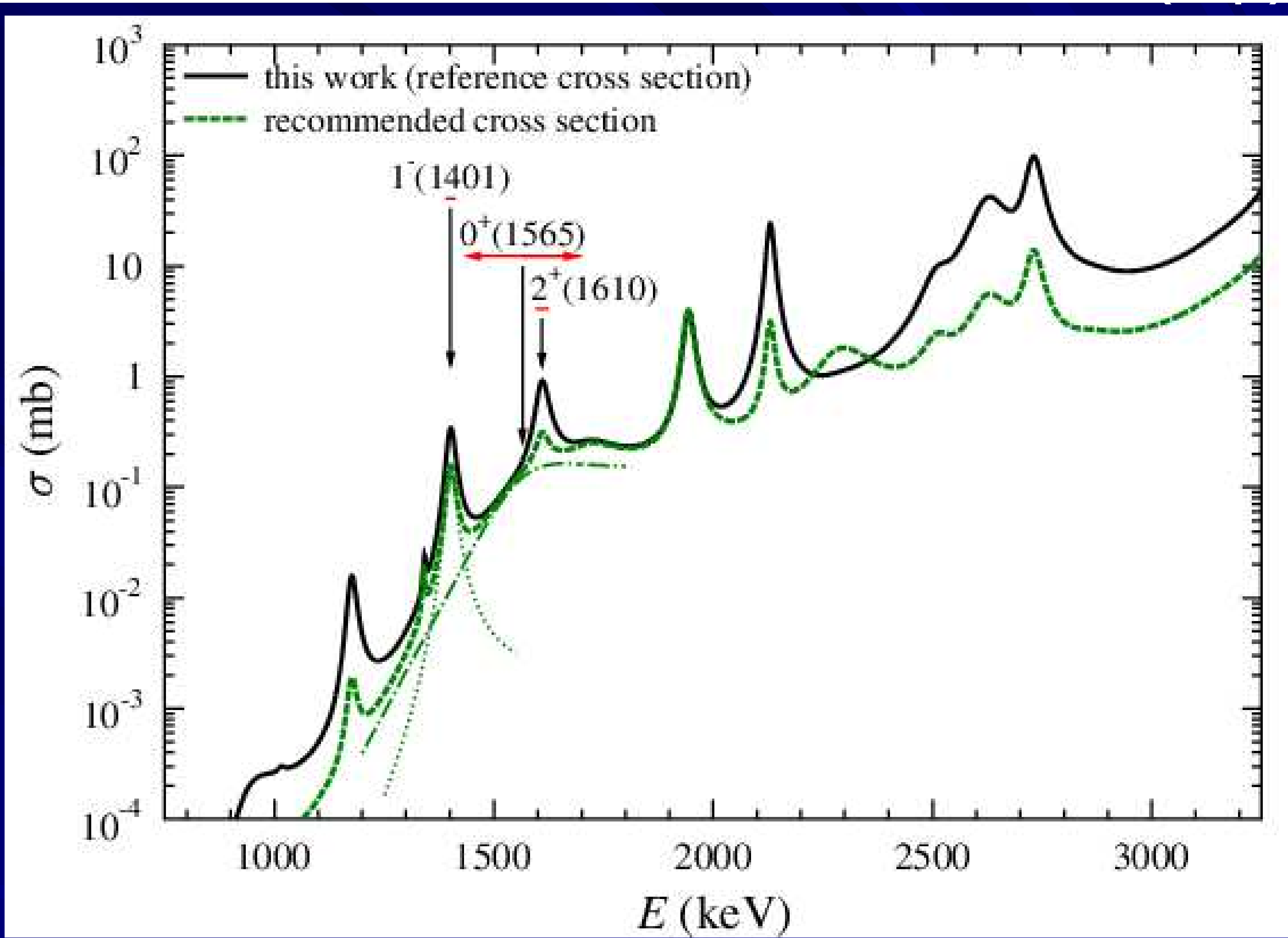
(same figure as before)

The recommended cross section for $^{18}Ne(\alpha,p)^{21}Na$



- Recommended cross section from new resonance strengths …

The recommended cross section for $^{18}Ne(\alpha,p)^{21}Na$



- … and the dominating resonances at 1401 keV and 1567 keV (not 1610 keV!); total widths Γ in red